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A Treatise of Gnomonics

or

DIALLING.

Done into *Engliſh*, and Amended in Several Places,
by *J. T. Deſaguliers* of *Hart-Hall*, OXON.

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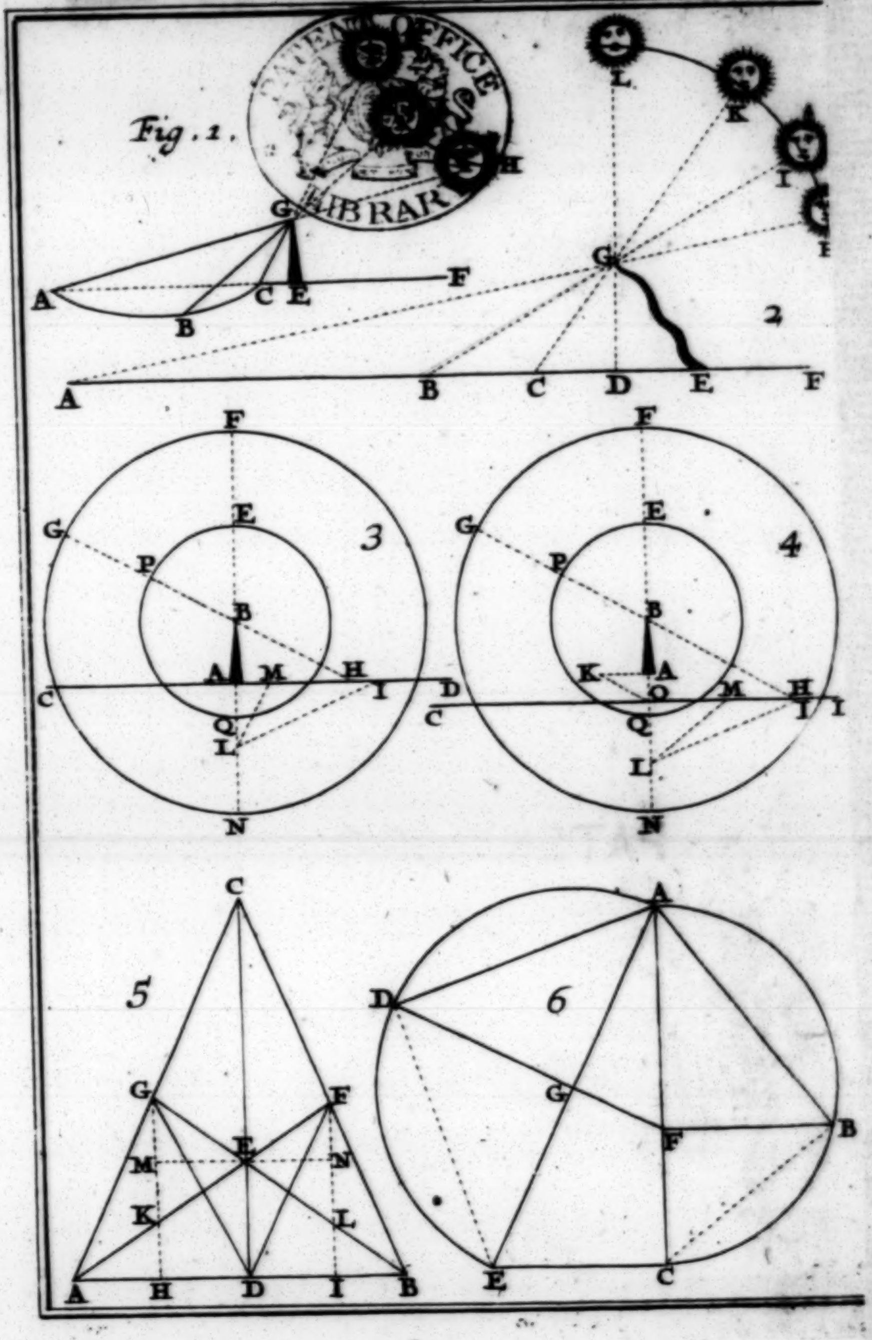
Gnomonicks or Dialling Plate 1 Pag. 1



A TREATISE OF *GNOMONICKS*.

GNOMONICKS, call'd alſo Dialling, is a Science, which, by means of the Rays of ſome Celeſtial Body, and eſpecially by means of the Rays of the Sun, divides the Time into equal parts, and repreſents upon a Surface almoſt the whole Machine of the *Primum Mobile*. It is a kind of Perſpective, where the Point of the Eye is in the Center of the Earth, and the Surface of the Dial repreſents the Picture, the Diſtance from the Eye to the Picture being equal to the length of the Style, which is always perpendicular to the Picture, and whoſe end conſequently repreſents the Point of the Eye, or the Center of the Earth.

Tho' we cannot ſtand in the Center of the Earth, to place there the Eye, or the end of the Style; yet the end of the Style rais'd perpendicularly upon the Plain of the Dial, which is on the Surface of the Earth, may be taken for the Center of the World, † or of the Earth; becauſe the ſmallneſs of the Diſtance from the Surface of the Earth to its Center, in compariſon of the Diſtance from the Sun to the Earth, can cauſe no ſenſible Error: And without placing the Eye at the end of the Style, the Rays of the Sun from different Points of the Heavens, do the Office of Viſual Rays, which wou'd meet the Plain of the Dial in the ſame place as the end of the Shadow of the Style falls upon it, and ſhew upon that Plain the Sun's place in the Heavens almoſt as exactly as if the end of the Style was in the Center of the Earth.

Imagine then a *Style* to be rais'd upon the plain AC, which Style, or Cock, is a little pointed Rod, fix'd upright upon the Plain where you wou'd draw a Dial, as EG, and the *Plate* I. *Fig.* 2.

N.B. † *The Author in this Treatise follows* Ptolemy*'s Syſtem.*

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Plate 1. *Fig.* 2. point G of that Style EG to be in the Center of the Earth, which we will ſuppoſe in the middle of the World, or at leaſt in the middle of the Celeſtial Circumvolutions ; ſo that the plain AF be diſtant from the Center of the Earth the whole length of the Style, which may be taken at pleaſure, becauſe a greater or a leſs Length cannot alter the exactneſs of the Dial, by reaſon of the vaſt Diſtance from the Sun to the Earth.

Imagine alſo, that from all the Points of the Sphere of the Sun, as H, I, K, L, *&c.* Rays be drawn thro' the Center of the Earth, or the end G of the Style EG, and produc'd till they meet the plain AF, at the points A,B,C,D, which will upon that plain AF give the Appearance or Repreſentation of the Points of the *Heaven*, or *Sphere* of the Sun from whence the ſaid Rays come : So that if the Center of the Sun was in each of thoſe points L,K,I,H, the Ray drawn from its Center thro' the end of the Style G, or rather the Shadow of that Style terminated by that Ray, wou'd fall upon the points A,B,C,D, in the Plain, which are the Repreſentation of the points L, K, I, H, or the different places of the Sun in its Heaven.

Therefore to draw a Sun-Dial upon a Plain propos'd, we will ſuppoſe the end of the Style the Center of the Earth : And tho' ſuch a Suppoſition be Mathematically Falſe ; yet the Dial will not be leſs True ; becauſe, as we have already ſaid, the Semi-Diameter of the Earth is very ſmall in compariſon of that of the Sphere of the Sun.

We ſhall alſo look upon the Diurnal and Nocturnal Arches of the Sun, as Circles parallel to one another and to the Equator ; for tho' theſe Arches are rather ſpiral Lines than true Circles, by reaſon of the continual and oblique Motion of the Sun about the Poles of the Zodiack, nevertheleſs becauſe the Sun moves very ſlowly along the Ecliptick, not going thro' One Degree in 24 Hours, you muſt ſuppoſe it to be a whole Day in the ſame Point of the Zodiack, and the Circle which it deſcribes in a Day from Eaſt to Weſt about the Poles of the World, muſt be look'd upon as parallel to the Equator.

Laſtly, in reſpect of the Hours, you may look upon the *Center of a Dial* deſcrib'd upon a Plain, which Center is the Point where all the Hour-lines meet, as the Center of the Earth, or the end of the Style ; becauſe the ſmall Diſtance between that Center and the end of the Style is but as a Point in reſpect of the Sphere of the Sun, ſo that the Error that may happen by reaſon of this falſe Suppoſition, cannot be conſiderable in reſpect of the *Horary-Angles*, that is, of the Angles which the *Hour-Lines* make with the Meridian in the Center of the Dial.

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COROLLARY I.

It follows from what we have ſaid, that the true *Length of* Plate 1.
the Style is not that Iron Rod which we ſee uſually rais'd on Fig. 2.
the Plain, unleſs it be perpendicular to the ſaid Plain: And when it is not ſo, as EG, the true length of that Style EG, muſt be expreſs'd by the Right-line GD drawn from the end of the Style G, perpendicular to the plain AF, and the point D where it meets the Plain will be the true Place of the Style, which is call'd the *Foot of the Style*, and that Perpendicular GD will repreſent its true Length. Its correſpondent point L over the Plain AF, is call'd the *Zenith of the Plain*, in which when the Sun is, the Shadow falls on the Foot of the Style D: And the Horizon which is parallel to the plain AF, is call'd the *Horizon of the Plain.*

COROLLARY II.

It follows alſo that a Right-line drawn thro' the Center of the Dial and the end of the Style, may be taken for the Axis of the World, and therefore it is uſually call'd the *Axis of the Dial*, becauſe the Axis of the World being an Imaginary Line which paſſes thro' the Poles of the World and the Center of the Earth, and the Center of the Dial being the Repreſentation of the Pole rais'd above the Plain, (namely, the Point where the end of the Shadow of the Style wou'd be terminated if the Sun was in that Pole) it follows that a Right-line drawn thro' the Center of the Dial, and the end of the Style which repreſents the Center of the Earth, may be taken for the Axis of the World.

COROLLARY III.

It follows alſo that the end of the Style may be taken for the Center of all the great Circles of the Sphere, becauſe all the great Circles of the Sphere have one common Center, *viz.* the Center of the Earth, which is in the Plain of all the great Circles of the Sphere: And as the end of the Style is taken for the Center of the Earth, it may be conſider'd as the Center of all the great Circles of the Sphere.

COROLLARY IV.

Laſtly, it follows, that all the great Circles of the Sphere muſt be conceiv'd as Plains, which cut one another at the end of the Style, becauſe it is their common Center: And as the Section of Two Plains is a Right-line, *by* 3.11. it follows,

Plate I. Fig. 2. that thoſe Plains will cut the Plains of the Dial in Right-lines; and ſince the end of the Style is in the Plain of thoſe great Circles, its Shadow will never go out of it, *by* I. II. and conſequently it can only be terminated in the common Section of thoſe Plains; that is, in a Right-line.

Thus you ſee, that *in* Gnomonicks *a great Circle of the Sphere is repreſented upon a Plain by a Right-line*; that is, as much as to ſay, that if the Sun ſhou'd move along the Circumference of the great Circle HIKL, (whoſe common Section with the Plain of the Dial is the line AF) going ſucceſſively thro' the points H,I,K,L, the Shadow of the end of the Style G will alſo go ſucceſſively thro' the points A, B, C, D, of the Right-line AF.

Fig. I. It is not ſo of a little Circle of the Sphere, as H,I,K, for as its Center is not the Center of the Earth, or in the end G of the Style EG, the Rays drawn from all the points H, I, K, of its Circumference thro' the end of the Style G are not in the ſame Plain, but they make the Right-Cone HGK, whoſe Vertex G, is at the end of the Style EG. Now as this Cone, which we ſhall call the *Cone of Light*, being produc'd beyond its Vertex G, makes another Cone ABCG, which ſhall be call'd the *Cone of Shadow*, that Cone of Shadow will be cut by AF the Plain of the Dial, along the Curve-line ABC, which is the Repreſentation of the little Circle HIK.

Thus you ſee, that *in* Gnomonicks *a little Circle of the Sphere is repreſented by a Curve Line*, which will always be ſome Conick Section, *viz.* a Circle, an Ellipſis, a Parabola, or a Hyperbola, according as the Cone of Shadow is more or leſs obliquely cut by the Plain of the Dial. But in the Practice it is needleſs to know of what kind the ſaid Curve is, becauſe it may be eaſily deſcrib'd upon the Plain of the Dial without knowing its Properties, as we ſhall ſhew in its proper place.

CHAPTER I.

Of the Lemma's.

TO make Practical Dialling the eaſier, we ſhall here add ſome Problems, and Theorems, to make plain all that we ſhall ſay concerning the Conſtruction of Dials, which by this means will be unmix'd with the Theory, which wou'd make it more difficult and leſs pleaſant.

LEMMA I.

THEOREM.

If a great Circle of the Sphere be perpendicular to the Plain of the Dial, it will be repreſented on it by a Right-line, which will go thro' the Foot of the the Style.

WE have already made it appear, that the Repreſentation of a great Circle of the Sphere upon a Plain is a Right-line: now I ſay, that That Right-line ought to go thro' the Foot of the Style; becauſe if it did not go thro' it, one might in the Plain of the great Circle let fall a Perpendicular from the end of the Style to the ſaid Line, which wou'd alſo be Perpendicular to the Plain of the Dial, *by Def.* 4. 11. and becauſe the length of the Style is alſo perpendicular to the Plain of the Dial it wou'd follow, that from One point Two Perpendiculars might be drawn to the ſame Plain, which is impoſſible.

COROLLARY.

It follows from this Propoſition, that the Repreſentation of a great Circle of the Sphere, which is not perpendicular to the Plain of a Dial, does not go thro' the Foot of the Style; and is nearer to or farther from it, according as the Plain of the Circle is more or leſs Inclin'd to the Plain of the Dial.

LEMMA II.

THEOREM.

If of Two great Circles of the Sphere, which are perpendicular to one another, the one be perpendicular to the Plain of the Dial; their Repreſentations will be Two Right-lines perpendicular to one another.

THE Demonſtration of this Theorem is plain; becauſe if thoſe Two Circles did not cut the Plain of the Dial by Two Right-lines perpendicular to one another, but thoſe lines were Inclin'd to one another, the Plains muſt be likewiſe Inclin'd to one another, and ſo not perpendicular, which is againſt the Suppoſition.

LEMMA III.

THEOREM.

A Right-line, which repreſents upon a Plain a great Circle of the Sphere, may be Geometrically divided into ſuch unequal Parts as ſhall repreſent the Degrees of that great Circle.

Plate 1. *Fig.* 3. LET B be the Center of the Earth, A the Foot of the Style, and AB its Length. Let FGH alſo be a great Circle of the Sphere, whoſe common Section with the Plain is the line CD which will be its Repreſentation. This being ſuppos'd, I ſay that the line CD may be Geometrically divided into Degrees, by means of a Point which we ſhall find in the Plain, as it wou'd be divided in the Air, by the Rays drawn from the Degrees of the Circle FGN, (which that line CD repreſents) thro' the Center of the Earth, or end of the Style B; as if the Arch FG, or the Angle ABH, was, *for Example*, of 60 Degrees, the Ray or *Radius* produc'd, GB, wou'd give upon the line CD, the part AH of 60 Degrees in Repreſentation. You wou'd have the ſame point H, if you made uſe of a little Circle concentrick to the Firſt, as as EPQ, becauſe the Arch ER is ſimilar to the Arch FG: Thus by means of the little Circle EPQ in the Air, you wou'd have the ſame part AH of 60 degrees in Repreſentation, which ſhews that one may Mechanically divide into degrees the line CD, by means of a little Circle, whoſe Center is at the end of the Style. But I ſay that ſuch a line may be Geometrically divided into degrees by means of a point, which being found upon the Plain of the Dial, will have the ſame Effect as the point B in the Air. All the difficulty therefore conſiſts in finding that point in the Plain of the Dial, which we ſhall call the *Dividing Center*, and which we muſt find in both the Caſes which may happen, becauſe the Line to be divided may go thro' the Foot of the Style, (*viz.* when it repreſents a Circle perpendicular to the Plain of the Dial) or elſe it may run at ſome diſtance from it, (when it repreſents a Circle whoſe Plain is Inclin'd to the Plain of the Dial, *by Lem.* 1.) and each Caſe will be thus reſolv'd.

Firſt Caſe.

IF the line to be divided goes thro' the Foot of the Style, as CD, draw thro' A the Foot of the Style the line AL, perpendicular to the line CD, and equal to the Style AB, and the

the point L will be the dividing Center of CD the line propos'd; ſo that if at the Point L the Angle ALI be made of *Plate* 1. *Fig.* 3. as many degrees as the Arch FG, or as the Angle ABH, by means of the Line LI, that Line LI will upon the Line CD give the ſame Point H.

DEMONSTRATION.

If the line LI being produc'd does not cut the line CD produc'd at the point H, let it, if poſſible, cut it in any other point as I, and then it will be known, *by* 26. 1. that the Two rectangular Triangles ABH, ALI, are equal, becauſe they have the Angles ABH, ALI, and the Sides AB, AL, equal, *by Conſtr.* Whence it follows that the ſides AH, AI, are alſo equal, which being impoſſible, it is alſo impoſſible for the line LI to meet the line CD in any other point but H. *Q. E. D.*

Second Caſe.

BUT if the line to be divided does not go thro' the Foot of the Style, as CD; to find its dividing Center L, draw *Fig.* 4. from A the Foot of the Style, to the line CD, the indefinite Perpendicular ON, which cuts it at the point O, and the Parallel AK equal to the Style AB, and join the Hypotenuſe OK, whoſe length being ſet off upon the Perpendicular ON, from O to L, that point L will be the Dividing Center requir'd; ſo that if at L you make the angle OLI equal to the angle ABH, by the line LI, that line LI will upon CD give the ſame point H.

DEMONSTRATION.

If you conceive the diſtance from the point in the Air B, to the point O upon the Plain, to be equal to the Hypotenuſe OK, (becauſe if the Triangle OAK was rais'd up at Right-angles upon the Plain of the Dial, the point K wou'd coincide with the end B of the Style AB, in which caſe the Hypotenuſe OK wou'd be perpendicular to the line CD, and coincide with the erected line OB, which is alſo perpendicular to CD;) we ſhall demonſtrate as before, that the Two Triangles OBH, OLI, are equal, whoſe Sides OB, OL, and Angles OBH, OLI, are equal, *by Conſtr.* &c.

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LEMMA IV.

THEOREM.

Plate I. Fig. 5.

If from the Two Ends A, B, of the Baſe AB, of the Triangle ABC, you draw thro' the point E (taken at pleaſure upon the perpendicular CD, which falls within the Triangle,) the lines AF, BG, and join the lines DF, DG; the Angle CDF will be equal to the Angle CDG, or the Angle ADG to the Angle BDF.

DRAW from the Two points F, G, the lines FI, GH, perpendicular to the Baſe AB, and thro' the point E, the Right-line MN parallel to the ſaid Baſe AB; and then you will eaſily know that the Two Rectangular Triangles BCD, BFI, are ſimilar, as well as the Two Obliquangular Triangles BEC, BLF; and likewiſe that the Two Rectangular Triangles ACD, AGH are ſimilar, as well as the Two Obliquangular Triangles AEC, AKG: and laſtly, that the Two Obliquangular Triangles ELF, EKG, are alſo ſimilar, as well as the Two Rectangular Triangles ENL, EMG.

This being ſuppos'd, you will in the Two ſimilar Triangles BCD, BFI, have this Analogy, BC: BF:: CD: FI; and in the Two ſimilar Triangles BCE, BFL, this, BC: BF:: CE: EL, from which, and the foregoing, is deduc'd this Third Analogy, CE: CD:: FL: FI. Likewiſe in the ſimilar Triangles ACE, AGK, you will have this, AC: AG: : CD: GH; and in the ſimilar Triangles ACE, AGK, this, AC: AG:: CE: GK, from which, and the foregoing, is deduc'd this Third Analogy, CE: CD:: GK: GH, from which, and the Third of the Three Firſt is deduc'd this, FL: FI:: GK: GH.

In the ſimilar Triangles ELF, EKG, you have this Analogy, EL: FL:: GE: GK, and in the Two ſimilar ENL, EMG, this, EN: EL:: EM: EG, from which, and the Laſt of the foregoing Paragraphs, is deduc'd, *by Equality*, this Laſt Analogy, EN: FI: : EM: GH, or DI: FI:: DH: GH; which ſhews, *by* 6.6. that the Angles FDI, GDH, are equal to one another, and conſequently the Two CDF, CDG. *Q. E. D.*

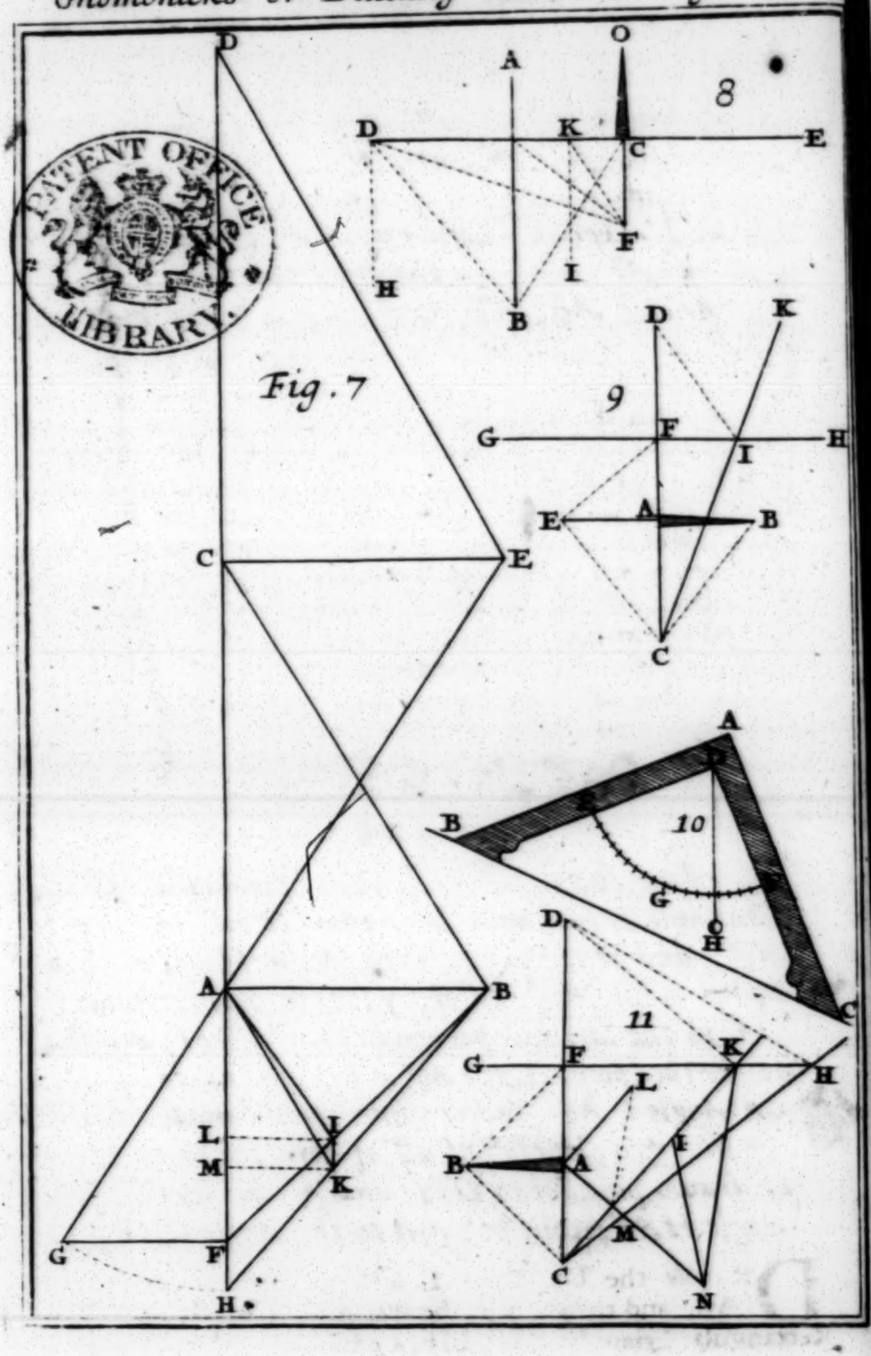
Fig. 7
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LEMMA V.

THEOREM.

Plate 1. Fig. 6.

If the Semi-circles ABC and ADE be deſcrib'd about the ſide AC, and the Hypotenuſe AE of the Triangle ACE, Rectangular at C; and from the point F taken at pleaſure upon AC to the Two lines AC and AE, be drawn the Perpendiculars FB, FD; the Chords AB, AD, will be equal to one another.

IF the Two Chords BC, DE, be drawn, the Two Angles ABC, ADE will be Right, *by* 31. 3. and *by* 8. 6. it will be known that the Two Rectangular Triangles ADE, ADG, are ſimilar; and *by* 4. 6. that the line AD is a Mean Proportional between the Two AE, AG; and laſtly, *by* 17. 6. that the ſquare of AD is equal to the Rectangle of the lines AE, AG. It will likewiſe be known after the ſame manner, that the ſquare of AB is equal to the Rectangle of the lines AC, AF. Laſtly, it will be known that in the ſimilar Triangles ACE, AFG, the Rectangle of the lines AE, AG, or the ſquare of AD, is equal to the Rectangle of the lines AC, AF, or to the ſquare of AB, and conſequently that the lines AB, AD, are equal to each other. *Q. E. D.*

LEMMA VI.

THEOREM.

Plate 1. Fig. 7.

If the Two lines AB, CE, be perpendicular to the ſame line DH, and CD is equal to BC, and AF to AB; and at the point A the Angle HAG be made equal to the Angle ACB, by the line AG terminated at G by the line FG, perpendicular to DH; and that at the ſaid point A, the Angle HAI be made equal to the Angle CAE, and the line AH be made equal to the line AG; and laſt of all, if thro' the point I, IK be drawn parallel to DH, and AK be joyn'd; the Angle HAK, will be equal to the Angle CDE.

DRAW the Two lines IL, KM, parallel each to the line AB, and then it will be eaſily known that the Two Rectangular Triangles ABF, LIF, are ſimilar, and conſequently that the Two lines LI, LF, are equal to one another, becauſe AB, and AF are equal, *by Conſtr.*

Plate 2. *Fig.* 7. In the ſimilar Triangles ABC, AFG, we have this Analogy. AC: AF:: BC: AG, and if inſtead of the Three Laſt Terms AF, BC, AG, we ſubſtitute the Three AB, CD, AH, which are equal to them, *by Conſtr.* we ſhall have this AC: AB:: CD: AH, and if inſtead of the Two Conſequents AB, AH, we put the Two MK, MH, which are in the ſame *Ratio*, becauſe of the ſimilar Triangles ABH, MKH, you will have this other Analogy, AC: MK:: CD: MH, which ſhews that the Rectangle of the Two lines AC, MH is equal to the Rectangle of the Two MK, CD, *by* 16. 6.

In the ſimilar Triangles ACE, ALI, one may know *by* 4. 6. that the *Ratio* of the Two lines AC, CE, is the ſame with that of the lines AL, LI, or AL, LF, becauſe LI, is equal to LF, or that of the Two BI, IF, by reaſon of the ſimilar Triangles ABF, LIF, or that of the Two BK, KH, by reaſon of the ſimilar Triangles BIK, BFH, or that of the Two AM, MH, by reaſon of the ſimilar Triangles ABH, MKH, and therefore, that AE: CE:: AM: MH; and *by* 16. 6. that the Rectangle of the Two lines AC, MH, or the Rectangle of the Two lines MK, CD, which has been demonſtrated equal to the foregoing, is equal to the Rectangle of the Two CE, AM, and conſequently, that the Triangle CDE is ſimilar to the Triangle AMK, and the Angle D equal to the Angle MAK. *Q. E. D.*

LEMMA VII.

How to draw the Horizontal-line upon a Plain.

Fig. 8. THE *Horizontal-line* is the Repreſentation upon a Plain, (or the common Section of that Plain, and) of the Plain of the Horizon. It muſt be a Right-line, becauſe the Horizon is a great Circle.

If the Plain be *Horizontal* or parallel to the Horizon, we ſhou'd endeavour in vain to find a Horizontal-line upon it, becauſe ſuch a Plain being ſuppos'd parallel to the Horizon, cannot be cut by the Plain of the ſaid Horizon, and conſequently can have no Horizontal-line.

But if the Plain propos'd be *Vertical*, that is, perpendicular to the Horizon, its Horizontal-line, *by Lemma* 1. goes thro' the Foot of the Style: and to find it, draw upon the Plain any Plumb-line, by means of a Plummet hanging freely by a Thread, as AB, and thro' the Foot of the Style C, draw DE perpendicular to this Line, and it will be the Horizontal-line requir'd.

DEMONSTRATION.

Plate 2. *Fig.* 8. The line AB repreſents a Vertical Circle, becauſe if it was produc'd *ad infinitum* on either ſide, it wou'd go directly to the *Zenith* and the *Nadir*, where the Vertical Circles interſect: and as that Vertical Circle, which ſoever it be, is perpendicular to the Horizon, and the Horizon perpendicular to the propos'd Plain, it follows *by Lemma* 2. that thoſe Two Circles are repreſented in the ſame Plain, by Two Lines perpendicular to one another; wherefore ſince the line AB repreſents one of thoſe Two Circles, *viz.* the Vertical, the other line DE, which is perpendicular to it, muſt repreſent the Horizon.

Fig. 9. If the Plain be *Inclin'd*, that is, if it cuts the Horizon at Oblique and Unequal Angles, let a Plummet and Thread hang from B the end of the Style, and where it touches the Surface of the Plain, mark a Point, as C, which will repreſent the *Zenith* or *Nadir*, according as that Surface is towards the Heavens, or the Earth. When it is towards the Sky, it is call'd *Upper Surface* or *Superficies*, where the *Zenith* falls below the Foot A of the Style; and when it is towards the Earth, it is call'd the *Under Surface*, where the *Nadir* happens to be above the Foot A of the Style.

This point C is call'd *Zenith* in the Upper Surface of the Plain, becauſe if the Sun was in the *Zenith*, the Shadow of the end of the Style B wou'd come to that point C: And *Nadir* in the Under Surface of the Plain, becauſe if the Sun was in the *Nadir*, and the Earth was tranſparent, the Shadow of B the end of the Style AB wou'd come to that point C; thro' which and the Foot A of the Style, you muſt draw the Right-line CD, which is uſually call'd the *Vertical of the Plain*, becauſe *by Lemma* 1. it repreſents a Vertical Circle perpendicular to the Plain, ſince it goes thro' A the Foot of the Style.

Laſtly, draw thro' A the Foot of the Style, the line AE, equal to the length of the Style AB, and perpendicular to the line CD, to have at E the Dividing Center of that line CD, *by Lemma* 3. Having joyn'd the Right-line EC, draw EF perpendicular to it, which will upon the line CD give the point F, thro' which you muſt draw to the ſaid line CD, the perpendicular GH, which will be the Horizontal-line requir'd.

DEMONSTRATION.

Becauſe the point E is the Dividing Center of the Vertical CD, and the Arch of each Vertical, comprehended between the *Zenith* or the *Nadir* and the Horizon, is of 90 Degrees, *Plate* 2. *Fig.* 9. we have alſo made the Angle CEF at that Center E, of 90 Degrees, to have the Arch CF of 90 Degrees in Repreſentation: And becauſe the point C here repreſents the *Zenith*, the point F will repreſent a Point of the Horizon, which being perpendicular to all the Vertical Circles, will be repreſented by the line GH, perpendicular to the line CD, *by Lemma* 2. Thus GH is the Horizontal-line. *Q. E. I. & D.*

SCHOLIUM.

The Angle ACE being meaſur'd with a Protractor, * or any other Inſtrument, gives the Inclination of the Plain; but it may be more eaſily and exactly known by means of a Square, having Two equal ſides as AB, AC, whoſe Angles B, C, muſt *Fig.* 10. be Half-Right, with a Quadrant or Fourth part of a Circle EGF within it, whoſe Center D muſt be at the Right-Angle A, and which muſt be divided into its 90 Degrees, beginning at its middle point G: For if the Upper Surface of the Inclin'd Plain be repreſented by the line BC, which is the Vertical-line of the Plain, by applying the ſquare to the ſaid Vertical as you ſee in the Figure, a Thread DH hanging freely with its Plummet H from the Center D, will ſhew upon the Quadrant EGF, (counting from the middle point G,) the Inclination of the Plain, which in this Example is of 25 Degrees.

* In French *this Inſtrument is* call'd Transporteur.

If the Inclination of the Plain is greater than 45 Degrees, you cannot know the Inclination of it by this Method, and therefore to find it, you muſt on the Vertical-line, or near its place if it is not drawn, apply a long ſtreight Ruler which has one part off of the Plain, that to the Under Part of the ſaid Ruler you may apply one of the ſides of the Square, *for Example*, the ſide AB, as if you work'd on the Under Surface of the Plain, and then the Thread and its Plummet hanging freely will ſhew the Inclination of the Plain, which muſt be counted from the other ſide AC, becauſe *Inclination of a Plain*, is the Angle which it makes with the Horizon.

LEMMA VIII.

PROBLEM.

How to find the Height of the Sun upon a Plain propos'd.

THE *Height of the Sun upon a Plain* is the ſame thing as the Height of the Sun above the Horizon of that Plain, which is equal to the Height of the Sun above the Horizon of the Place where you are, when the Plain propos'd is Horizontal, becauſe the Horizon of that Plain is the ſame with the Horizon of the Place. This Height may be found upon any Plain by means of a Point of Shadow, thus. *Plate* 2. *Fig.* 9.

Let the end of the Shadow of the Style AB be at C, and let it be requir'd at ſuch a Time to find the Height of the Sun on that Plain. Draw thro' the Point A of the Style, and thro' that Point of Shadow C, the line AC, at the ſaid Foot of the Style draw to AC the Perpendicular AE equal to the length of the Style AB, to joyn CE; which will give the Angle ACE, which is the Height of the Sun above that Plain.

DEMONSTRATION.

The Foot A of the Style repreſents the Zenith of the Plain, and the Point of Shadow C repreſents the place of the Sun in the Plain, wherefore the line AC will repreſent the Vertical of the Sun in reſpect to the Horizon of the Plain, and the part AC will repreſent the diſtance from the Sun to the *Zenith* of the Plain, or the Complement of the Height of the Sun above the Plain: And as that Arch AC is meaſur'd by the Angle AEC, *by Lemma* 3. becauſe the point E is the Dividing Center of the line AC, it follows that the Complement of the Angle AEC, (*viz.* the Angle ACE) will be the Height of the Sun above the Plain, or above the Horizon of the Plain. *Q. E. I. & D.*

LEMMA IX.

PROBLEM.

How to find the Height of the Sun above the Horizon.

WE ſhall alſo find this Height by means of a point of Shadow mark'd on any Plain whatever: And firſt, if that Plain be Horizontal, having *by Lemma* 8. found the Height of the Sun above that Plain, you will have the Height of the Sun above the Horizon, becauſe the Horizon of the Plain is only the Horizon of the place where the Plain is.

But if the Plain be Vertical, having mark'd upon it the point B of the Shadow of the end O of the Style OC, draw thro' that point B the Plumb-line AB, which will repreſent the Vertical of the Sun, whoſe place is repreſented on the Plain by the point B: And having mark'd upon the Plain the Horizontal-line DE, *by Lemma* 7. make GD equal to GF, after you have drawn CF equal to the length of the Style CO, and perpendicular to the Horizontal-line DE, and draw *Fig.* 8.

draw DB; and the Angle GDB will be the Height of the Sun above the Horizon.

DEMONSTRATION.

Plate 2. *Fig.* 8. Becauſe the line AB repreſents the Vertical of the Sun, *by Lemma* 7. and the point B is the place of the Sun in the Plain, the part BG will repreſent the diſtance from the Sun to the Horizon, or the Height of the Sun above the Horizon: And becauſe the point D is the Dividing Center of the Arch GB, *by Lemma* 3. that part GB will be meaſur'd by the Angle GDB, which conſequently will be the Height of the Sun above the Horizon. *Q. E. I. & D.*

Fig. 11. If the Point of Shadow of the end B of the Style AB be mark'd upon an Inclin'd Plain, as I, having mark'd upon that Plain the *Zenith* or *Nadir* C, and the Horizontal-line GH, *by Lemma* 7. draw from the *Zenith* or *Nadir* C, thro' the point I, the line CK, which will repreſent the Vertical of the Sun. Then from the Foot of the Style A, draw AL parallel to CK and equal to the length of the Style AB, and the Indefinite Perpendicular AN, which will be terminated at N, (if you make MN equal to ML,) and then draw the lines NK, NI, whoſe Angle INK will be the Height of the Sun requir'd.

DEMONSTRATION.

It is evident that the line CK, which goes thro' the *Zenith* or *Nadir* C, and the place of the Sun I in the Plain, is the Vertical of the Sun, and conſequently the Part IK repreſents the Height of the Sun above the Horizon: And as that part IK is meaſur'd by the Angle INK, becauſe the point N is the dividing Center of the line CK, *by Lemma* 5. it follows, That that Angle INK is the Height of the Sun above the Horizon. *Q. E. I. & D.*

LEMMA X.

PROBLEM.

How to find the Sun's Declination, by knowing its diſtance from the neareſt Equinoctial Point.

Plate. 3. *Fig.* 12. IMagine a Plain parallel to the Equator, and chooſe upon it a point, as A, for the Foot of the Style, or for One of the Poles of the World. Draw thro' that point A, the Indefinite Line AD, which you muſt look upon as the *Colure* of the Solſtices; and draw to it thro' the ſame point A the Perpendicular

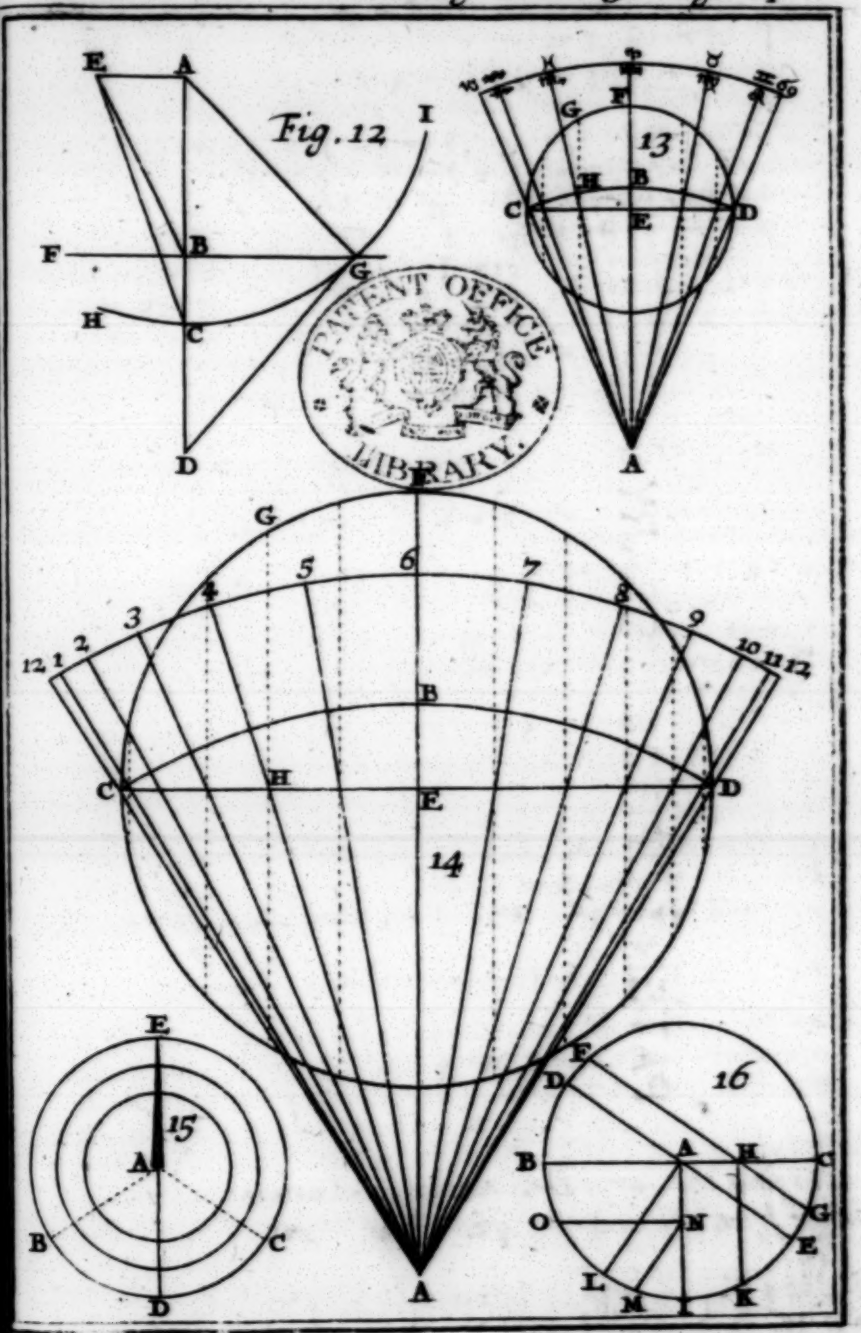
Fig. 12
E
A
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♐ ♏ ♎ ♍ ♌
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12 1
10 11 12
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E
D
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15
E
A
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16
F
D
B
A
H
C
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E
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K

dicular AE of what length you pleaſe, which you muſt take for the length of the Style, and then the point E will be the Dividing Center of the line AD, *by Lemma* 3. Draw to that dividing Center E, the Angle AEB of 66 degrees and a half, (that is, the Complement of the Sun's greateſt Declination which is 23 degrees and a half,) and draw thro' the point B, the line FG perpendicular to the line AD. Set off BE on BD, and at the point D make the Angle ADG equal to the Complement of the given Diſtance from the Sun to the neareſt Equinoctial Point. Laſtly, draw from the point A, for your Center, thro' the point G, the Arch HGI, and thro' the point C, where it cuts the line AD, draw the line EC, and the Angle ACE will be the Declination requir'd. *Plate* 3. *Fig.* 12.

DEMONSTRATION.

Becauſe the point A repreſents the Pole of the World, and the Arch AB of the Colure of Solſtices is of $66\frac{1}{2}$ degrees in Repreſentation, the point B is the Solſtitial Point, and the Perpendicular FG will repreſent the Ecliptick, *by Lemma* 2. whoſe Dividing Center being the point D, the point G will repreſent the Sun's place in the Ecliptick, and the Arch HGI will repreſent it Parallel: For the Parallels of the Sun are repreſented upon a Plain parallel to the Equator by Circles, becauſe in that Caſe the Cone of Shadow is cut by a Plain parallel to its Baſe. Thus the part AG or AC repreſents the diſtance of the Sun from the Pole, or the Complement of the Declination of the Sun; and as that part AC is meaſur'd by the Angle AEC, its Complement or the Angle ACE will conſequently be the Sun's Declination. *Q. E. I. & D.*

SCHOLIUM.

If you wou'd make uſe of the common Method, which depends upon the Principles of the Orthographical Projection of the Sphere, draw the line AB of what length you will, and from its end A draw at pleaſure the Arch CBD, to take upon it from B on either ſide the Arches BC, BD, of 23 degrees and a half each, for the Sun's greateſt Declination; and joyn the Right-line CD, which will be divided at Right-angles and into Two equal parts at E by the Right-line AB. Draw from that point E, thro' the Two points C, D, the Circumference FCD, to take upon it the Arch FG of the given diſtance of the Sun from the neareſt Equinoctial Point, and draw from the point G, the line GH perpendicular to the Diameter CD. Laſtly, draw from the point A, thro' the point H, where the Perpendicular *Fig.* 13.

Fig. 13. pendicular GH cuts the Arch CBD, the Right-line AH; and the Angle BAH, or the Arch BH, will be the Declination of the Sun requir'd.

Hence it follows, That if you divide the whole Circumference FCD, into parts of 30 degrees, or into Twelve equal parts for the 12 Signs of the Zodiack, and end the reſt as has been taught, you will make an Inſtrument, uſually call'd a *Triangle of the Signs,* which is very proper to draw upon a Dial the Arches or Parallels of the Signs, as we ſhall teach in *Chap* 3.

Plate 3. *Fig.* 12. The Declination of the Sun may be known more exactly by Trigonometry, in the Rectangular Spherical Triangle ABG, in which beſides the Right-angle B, you know the ſide AB, or the Complement of the Obliquity of the Zodiack, and the ſide BG, or the Complement of the diſtance of the Sun from the neareſt Equinoctial Point. Wherefore to know the Hypotenuſe AG, or the Complement of the Sun's Declination, you muſt make uſe of this Analogy.

As the Whole Sine:
To the Sine of the Obliquity of the Zodiack ::
So the Sine of the diſtance from the Sun to the neareſt Equinoctial Point.
To the Sine of the Sun's Declination;

The following Table has been calculated according to this Method, which Table ſhews the Declination of all the Degrees of the Signs of the Zodiack, the greateſt being ſuppos'd of 23 Degrees and 30 Minutes, tho' now it is but of about 23 Degrees and 29 Minutes, the difference of One Minute being inſignificant in Dialling. The Degrees, whoſe Number encreaſes as you go downwards in the Firſt Column towards the Left, are for the Upper Signs: and the Degrees, whoſe Number decreaſes as you go downwards in the Laſt Column towards the Right Hand, are for the Lower Signs.

A Table of the Declination of all the Degrees of the Ecliptick.

S	♈ ♎	♉ ♏	♐ ♊	S.
D.	D. M.	D. M.	D. M.	D.
1	0. 24	11. 51	20. 25	29
2	0. 48	12. 12	20. 36	28
3	1. 12	12. 32	20. 48	27
4	1. 35	12. 53	21. 0	26
5	2. 0	13. 13	21. 11	25
6	2. 23	13. 33	21. 21	24
7	2. 47	13. 53	21. 32	23
8	3. 11	14. 12	21. 42	22
9	3. 35	14. 32	21. 51	21
10	3. 58	14. 51	22. 0	20
11	4. 22	15. 9	22. 8	19
12	4. 45	15. 28	22. 17	18
13	5. 9	15. 47	22. 24	17
14	5. 32	16. 5	22. 32	16
15	5. 55	16. 22	22. 39	15
16	6. 19	16. 40	22. 46	14
17	6. 42	16. 57	22. 52	13
18	7. 5	17. 14	22. 57	12
19	7. 28	17. 30	23. 2	11
20	7. 50	17. 47	23. 7	10
21	8. 13	18. 3	23. 11	9
22	8. 35	18. 18	23. 15	8
23	8. 58	18. 34	23. 18	7
24	9. 20	18. 49	23. 21	6
25	9. 42	19. 3	23. 24	5
26	10. 4	19. 18	23. 26	4
27	10. 26	19. 32	23. 27	3
28	10. 47	19. 46	23. 28	2
29	11. 9	19. 59	23. 29	1
30	11. 30	20. 12	23. 30	0
S.	♓ ♍	♒ ♌	♑ ♋	S.

Degrees of the Upper Signs.

Declinations.

Declinations.

Degrees of the Lower Signs.

LEMMA XI.

PROBLEM.

How to find the Sun's Declination, if it ſhou'd ſet or riſe at a given Hour, for a given Latitude.

Plate 3. *Fig.* 12. SUppoſe a Plain to be parallel to the Equator, and chooſe in it a Point as A, for the Foot of the Style, or for One of the Poles of the World. Draw thro' that Point A, the Indefinite Line AD, which you muſt make the Meridian of the Place, and draw thro' the ſaid Point A, AE perpendicular to it, and of what Length you pleaſe, which you muſt take for the Length of the Style, and then the point E will be the Dividing Center of the line AD, *by Lemma* 3. At that Dividing Center E make the Angle AEB of the Height of the Pole above the Horizon, and draw thro' the point B the line FG, perpendicular to the line AD. At the point A make the Angle DAG of the given Horary Diſtance, taking 15 Degrees for One Hour, 30 for Two, *&c.* Laſtly, deſcribe from the point A, for your Center, thro' the point G, the Arch HGI, and thro' the point C, where it cuts the line AD, draw the line EC, and the Angle ACE will be the Declination requir'd.

DEMONSTRATION.

Becauſe the point A repreſents the Pole of the World, and the Arch AB of the Meridian is equal to the Elevation of the Pole above the Horizon, the point B will be in the Horizon, and the Perpendicular FG will repreſent the Horizon, *by Lemma* 2. and becauſe the Angle DAG is equal to the Sun's Horary diſtance, the line AG will repreſent the Hour-Circle of the Sun, and the point G will be the riſing or ſetting Point of the Sun, wherefore the Circle HGI will repreſent the Parallel of the Sun, as you read *in Lemma* 10. where you ſaw alſo that the Angle ACE is the Sun's Declination. *Q. E. I. & D.*

SCHOLIUM.

Fig. 14. If you wou'd make uſe of the common Method, which depends upon the Orthographical Projection of the Sphere, draw the line AB of what Length you pleaſe, and from its end A, draw at pleaſure the Arch CBD, to take upon it from B (on either ſide) the Two Arches BC, BD, each of the Com-

Complement of the Elevation of the Pole above the Horizon, and joyn the line CD, which will be divided at Right Angles and into Two equal parts at the point E, by the line AB. Draw from that point E, thro' the Two points C, D, the Circumference FCD, to take upon it the Arch CG of the Horary Diſtance, and draw from the point G, GH perpendicular to the Diameter CD. Laſtly, draw from A, thro' the point H where the Perpendicular GH cuts the Diameter CD, the line AH; and the Angle BAH, will be the Declination which the Sun wou'd have, if it ſhou'd Riſe or Set at the Hour propos'd. *Plate 3. Fig. 14.*

It follows, That if the Whole Circumference FCD be Divided into Parts of 15 Degrees each, or into 24 equal parts, and the reſt be ended as has been taught, an Inſtrument will be made, which is uſually call'd the *Triangle of Diurnal and Nocturnal Arches*, which is very uſeful to draw upon a Dial the Parallels of the Riſing or Setting of the Sun, or the Length of the Days and Nights, as will be taught in *Chap.* 3.

This Declination may be known more exactly by Trigonometry in the Rectangular Spherical Triangle ABG, in which beſides the Right Angle B, the ſide AB (or the Complement of the Sun's greateſt Declination) is known, and alſo the Angle BAG, which is the Horary Diſtance. Wherefore to know the Hypotenuſe AC, or the Complement of the Sun's Declination, you muſt make this Analogy: *Fig. 12*

As the Whole Sine :
To the Sine of the Complement of the Horary Diſtance : :
To the Tangent of the Complement of the Height of the Pole :
To the Tangent of the Sun's Declination.

After this manner the following Table has been Calculated, which ſhews the Declination of the Diurnal and Nocturnal Arches for every Hour for the different Height of the Pole from 30 to 60 Degrees.

A Table of the Declination of the Diurnal and Nocturnal Arches for different Latitudes.

A	XIII.	XIV.	XV.	XVI.	XVII.	XVIII.
P	D.M.	D.M.	D.M.	D.M.	D. M.	D. M.
30	12.44	24 9	33.32	40.54	46. 31	50. 46
31	12.16	23.18	32.29	39.46	45. 22	49. 39
32	11.48	22.30	31.29	38.40	44. 15	48. 32
33	11.22	21.44	30.31	37.46	43. 25	47. 26
34	10.57	21. 0	29 34	36.33	42. 4	46. 21
35	10.34	20 17	28.40	35.32	41. 0	45. 17
36	10.11	19.36	27.47	34.32	39. 58	44. 13
37	9.50	18.57	26.55	33.34	38. 56	43. 11
38	9.29	18.20	26. 6	32.37	37. 55	42. 9
39	9. 9	17.43	25.18	31.42	36. 56	41. 8
40	8.50	17. 9	24.31	30.47	35. 38	40. 7
41	8.31	16.35	23.46	29.55	35. 0	39. 8
42	8.15	16. 2	23. 1	29. 3	34. 4	38. 9
43	7.58	15.31	22.19	28.12	33. 8	37. 10
44	7.42	15. 0	21.37	27.22	32. 14	36. 13
45	7.20	14.31	20.56	26.34	31. 20	35. 16
46	7.11	14. 2	20.17	25.46	30. 17	34. 20
47	6.56	13.34	19.38	25. 0	29. 35	33. 24
48	6.42	13. 7	19. 1	24.14	28.44	32. 29
49	6.28	12.41	18.24	23.30	27. 53	31. 35
50	6.15	12.15	17.48	22.46	27. 4	30. 41
51	6. 2	11.50	17.13	22. 3	26. 15	29. 48
52	5.49	11.26	16.39	21.20	25. 26	28. 55
53	5.37	11. 2	16. 6	20.39	24. 39	28. 3
54	5.25	10.39	15.33	19 58	23. 52	27. 12
55	5.13	10.17	15. 0	19.17	23. 5	26. 21
56	5. 2	9.54	14 29	18 38	22. 20	25. 30
57	4.51	9.33	13 57	17.59	21. 34	24. 40
58	4.40	9.11	13.27	17.21	20. 50	23. 50
59	4 29	8.50	12.57	16.44	20. 5	23. 1
60	4.19	8.30	12.28	16. 6	19. 22	22. 12

A Con-

A	XIX.		XX.		XXI.		XXII.		XXIII		XXIV.	
P	D.	M.	D.	M	D.	M	D.	M.	D.	M.	D.	M.
30	53.	57	56.	18	58.	0	59.	8	59.	47	60.	0
31	52.	50	55.	15	56.	58	58.	7	58.	47	59.	0
32	51.	46	54.	11	55.	56	57.	6	57.	47	58.	0
33	50.	42	53.	8	54.	54	56.	5	56.	46	57.	0
34	49.	38	52.	5	53.	52	55.	4	55.	46	56.	0
35	48.	34	51.	3	52.	50	54.	4	54.	46	55.	0
36	47.	31	50.	0	51.	49	53.	3	53.	46	54.	0
37	46.	28	48.	58	50.	48	52.	2	52.	46	53.	0
38	45.	26	47.	57	49.	47	51.	2	51.	46	52.	0
39	44.	25	46.	55	48.	46	50.	1	50.	45	51.	0
40	43.	24	45.	54	47.	45	49.	2	49.	45	50.	0
41	42.	23	44.	54	46.	45	48.	1	48.	45	49.	0
42	41.	23	43.	53	45.	44	47.	1	47.	45	48.	0
43	40.	23	42.	53	44.	44	46.	0	46.	45	47.	0
44	39.	24	41.	53	43.	44	45.	0	45.	45	46.	0
45	38.	25	40.	54	42.	44	44.	1	44.	45	45.	0
46	37.	28	39.	54	41.	44	43.	0	43.	46	44.	0
47	36.	30	38.	55	40.	45	42.	1	42.	45	43.	0
48	35.	33	37.	57	39.	45	41.	1	41.	45	42.	0
49	34.	36	36.	58	38.	46	40.	1	40.	45	41.	0
50	33.	39	36.	0	37.	47	39.	1	39.	45	40.	0
51	32.	43	35.	3	36.	48	38.	2	38.	46	39.	0
52	31.	48	34.	5	35.	49	37.	2	37.	45	38.	0
53	30.	52	33.	8	34.	51	36.	3	36.	46	37.	0
54	29.	58	32.	11	33.	52	35.	4	35.	46	36.	0
55	29.	3	31.	14	32.	54	34.	4	34.	46	35.	0
56	28.	9	30.	18	31.	56	33.	5	33.	46	34.	0
57	27.	16	29.	21	30.	58	32.	6	32.	46	33.	0
58	26.	22	28.	25	30.	0	31.	7	31.	47	32.	0
59	25.	29	27.	30	29.	3	30.	8	30.	46	31.	0
60	24.	37	26.	34	28.	6	29.	9	29.	45	30.	0

If you wou'd know the Quantity of the Horary Arch BG ſet off the Hypotenuſe BE on BD, and draw the Right-line

Plate 3. *Fig.* 12. DG, and the Angle ADG will be the Quantity of the Horary Arch BG, which may be found more exactly by Trigonometry, by the following Analogy in the ſaid Rectangular Triangle ABG:

As the Whole Sine:
To the Sine of the Elevation of the Pole: ;
So the Tangent of the Horary Diſtance:
To the Tangent of the Horary Arch.

According to this Method we have calculated the following Table, which ſhews the Horary Arches of every Quarter of an Hour, for every Degree of Latitude. Theſe Marks (*) ſhew the Half-Hours, between which, and the Whole Hours, are the Quarters, without any Mark over them.

A Tr-

A Table of the Horary Arches, for evnry Degree of Latitude.

H	.	*		I.XI.		*
P	D.M	D M	D.M.	D.M.	D.M	D.M.
1	0. 4	0. 8	0.12	0.16	0.20	0.25
2	0. 8	0.16	0.24	0.32	0.41	0.50
3	0.12	0.24	0.36	0.48	1. 1	1.15
4	0.16	0.32	0.48	1. 4	1.21	1.39
5	0.20	0.39	1. 9	1.20	142	2. 4
6	0.24	0.47	1.11	1.36	2. 2	2.29
7	0 28	0.55	1.23	1.52	2.22	2.53
8	0.31	1. 3	1.35	2. 8	2.42	3.18
9	0.36	1.11	1.47	2.21	3. 2	3.42
10	0.39	1.19	1.59	2.40	3.22	4. 7
11	0.44	1.26	2.10	2.55	3.43	4.31
12	0.47	1.34	2.22	3.11	4. 2	4.55
13	0.51	1.42	2.34	3.27	4.22	5.20
14	0.55	1.49	2.45	3 43	4.41	5.43
15	0.59	1.57	2.57	4.58	5. 1	6. 7
16	1. 2	2. 5	3. 8	4.14	5.21	6.31
17	1. 6	2.12	3.20	4.29	5.40	6.54
18	1. 9	2 20	3.31	4.44	5.59	7.18
19	1.13	2.28	3.42	4.59	6.18	7.41
20	1.17	2.35	3.54	5 16	6.37	8. 4
21	1.21	2.42	4. 5	5.29	6.56	8.27
22	1.25	2.50	4.16	5.46	7.15	8.49
23	1.28	2.57	4 27	5.59	7.33	9 12
24	1.32	3. 4	4.38	6.13	7.52	9:34
25	1.35	3.11	4.48	6.30	8.10	9 56
26	1.39	3.18	4.59	6.42	8.28	10.17
27	1.42	3.25	5.10	6 56	8.46	10.39
28	1.46	3.32	5.20	7.14	9. 3	11. 0
29	1.49	3.39	5.31	7.28	9 21	11.21
30	1 53	3 46	5.41	7.38	9.38	11.42

A Continuation of the Table.

H				I. XI.		
P	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
31	1.56	3.53	5.51	7 52	9.55	12. 3
32	1.59	4. 0	6. 1	8. 5	10.12	12.23
33	2. 3	4. 7	6.11	8.18	10.28	12.42
34	2. 6	4.14	6.21	8.31	10 45	13. 2
35	2. 9	4.20	6.32	8.44	11. 1	13.22
36	2.12	4.26	6.40	8.57	11.18	13.41
37	2.16	4.32	6.50	9.10	11.33	14. 0
38	2.19	4.38	6.59	9.22	11.48	14.18
39	2.22	4.44	7. 8	9.34	12. 4	14 37
40	2.25	4.50	7.17	9.47	12.19	14.54
41	2.28	4.56	7.26	9.58	12.33	15.13
42	2.31	5. 2	7.35	10. 7	12.48	15.29
43	2.34	5. 8	7.43	10.21	13. 2	15.46
44	2.36	5.14	7.52	10.33	13 16	16. 3
45	2.39	5.20	8. 0	10.44	13.30	16.20
46	2.42	5.25	8. 9	10 55	13.44	16.35
47	2.45	5.30	8.17	11. 6	13.56	16.51
48	2.47	5.35	8.25	11.16	14. 9	17. 6
49	2.50	5.40	8.32	11.26	14.22	17.22
50	2.52	5.45	8.40	11.36	14.35	17.37
51	2.55	5.50	8.47	11.46	14.47	17.51
52	2.57	5.55	8.54	11.56	14.59	18. 5
53	3. 0	6. 0	9. 1	12. 5	15.10	18.18
54	3. 2	6. 5	9. 8	12.14	15.21	18.31
55	3. 4	6. 9	9.15	12.23	15.33	18.45
56	3. 7	6.14	9.22	12.32	15.45	18.58
57	3. 9	6.18	9.28	12.40	15.53	19. 9
58	3.11	6.25	9.34	12.48	16. 3	19.22
59	3.13	6.30	9.40	12.56	16.13	19.33
60	3.15	6.34	9.46	13. 4	16 23	19.44

A Continuation of the Table.

H		*		I. XI.		*
P	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
61	3.17	6.38	9.52	13.12	16.32	19.55
62	3.19	6.42	9.58	13.20	16.42	20. 7
63	3.21	6.45	10. 3	13.26	16.50	20.15
64	3.23	6.48	10. 8	13.33	16.58	20.25
65	3.24	6.51	10.13	13.40	17. 6	20.35
66	3.26	6.54	10.18	13.46	17.14	20.44
67	3.27	6.57	10.23	13.52	17.21	20.52
68	3.28	7. 0	10.27	13.57	17.28	21. 0
69	3.30	7. 3	10.31	14. 3	17.35	21. 8
70	3.31	7. 6	10.35	14. 8	17.42	21.16
71	3.33	7. 8	10.39	14.13	17.48	21.23
72	3.34	7.10	10.43	14.18	17.54	21.29
73	3.35	7.12	10.46	14.23	17.59	21.36
74	3.36	7.13	10.50	14.27	18. 4	21.43
75	3.37	7.15	10.53	14.31	18. 9	21.49
76	3.38	7.17	10.55	14.35	18.14	21.54
77	3.39	7.18	10.58	14.38	18.18	21.59
78	3.40	7.20	11. 1	14.42	18.22	22. 3
79	3.41	7.22	11. 5	14.44	18.24	22. 7
80	3.42	7.23	11. 6	14.47	18.26	22.11
81	3.42	7.25	11. 7	14.49	18.32	22.15
82	3.43	7.26	11. 9	14.52	18.35	22.18
83	3.44	7.27	11.10	14.54	18.37	22.21
84	3.44	7.28	11.12	14.55	18.39	22.23
85	3.45	7.29	11.13	14.57	18.41	22.25
86	3.45	7.29	11.13	14.58	18.42	22.27
87	3.45	7.30	11.14	14.59	18.43	22.28
88	3.45	7.30	11.15	15. 0	18.44	22.29
89	3.45	7.30	11.15	15. 0	18.45	22.30
90	3.45	7.30	11.15	15. 0	18.45	22.30

A Con-

A Continuation of the Table.

H		II.X.		*		III.IX
P	D.M.	D.M.	D.M.	D.M.	D. M.	
1	0.30	0.35	0.40	0.46	0.53	1. 0
2	0.59	1. 9	1.20	1.32	1.45	2. 0
3	1.29	1.44	2. 0	2.18	2.38	3. 0
4	1.58	2.19	2.44	3. 4	3.30	3.59
5	2.27	2.52	3.20	3.49	4 22	4.59
6	2.57	3.27	4. 0	4.35	5.14	5.58
7	3.26	4. 1	4.39	5.20	6. 6	6.57
8	3.55	4.36	5.19	6. 6	6.57	7.55
9	4.25	5.10	5.58	6.51	7.49	8.54
10	4.54	5.44	6.57	7.35	8.40	9 51
11	5.23	6.17	7.16	8.20	9.30	10.48
12	5.51	6.51	7.55	9. 4	10.20	11.45
13	6.20	7.24	8.33	9.48	11.10	12.41
14	6.48	7.57	9.11	10.31	11.59	13.36
15	7.17	8.30	9.48	11,14	12.48	14.31
16	7.42	9. 3	10.26	11.57	13.36	15.25
17	8.12	9.35	11. 3	12.38	14.23	16.18
18	8.40	10.11	11 40	13.20	15.10	17.10
19	9. 7	10.39	12.16	14. 2	15.56	18. 2
20	9.34	11.10	12.52	14 43	16.42	18 53
21	10. 2	11.41	13.27	15.23	17.27	19.43
22	10.28	12.13	14. 2	16. 2	18.11	20.32
23	10.54	12.43	14.38	16. 41	18.55	21.20
24	11.20	13.13	15.12	17.20	19.38	22. 8
25	11.46	13.43	15 46	17.58	20.20	22.54
26	12.12	14.11	16.19	18 35	21. 2	23.40
27	12.37	14.42	16.53	19.12	21.42	24.25
28	13. 2	15.10	17.25	19.49	22.23	25. 9
29	13.27	15.38	17.57	20 24	23. 2	25.52
30	13.50	16. 6	18.28	20.59	23.41	26.34

A Continuation of the Table.

H		II. X.		*		III.IX.
P	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
31	14.15	16.34	19. 0	21.34	24.19	27. 5
32	14.39	17. 1	19.30	22. 9	24.56	27.55
33	15. 2	17.28	20. 0	22.40	25.31	28.34
34	15.25	17.54	20.29	23.14	26. 7	29.13
35	15.48	18.19	20.57	23.45	26.42	29.50
36	16.10	18.45	21.26	24.16	27.26	30.27
37	16.32	19. 9	21.54	24.47	27.49	31. 2
38	16.53	19.34	22.21	25.17	28.22	31.37
39	17.14	19.58	22.49	25.46	28.54	32. 11
40	17.35	20.21	23.15	26.15	29.25	32.44
41	17 56	20.45	23.41	26.43	29.55	33.16
42	18.16	21. 7	24. 6	27.11	30.24	33.47
43	18.35	21.29	24.30	27.37	30.53	34 18
44	18.55	21.51	24.54	28. 4	31.21	34.47
45	19.13	22.12	25.17	28.25	31.48	35.16
46	19.32	22.33	25.40	28.54	32. 9	35.44
47	19.46	22.53	26. 3	29.18	32.41	36.11
48	20. 8	23.13	26.24	29.42	33. 7	36.37
49	20.25	23.33	26.45	30. 4	33.30	37. 3
50	20.42	23.52	27. 6	30.27	33.54	37.27
51	21. 0	24 10	27.26	30.48	34.17	37.51
52	21.15	24.28	27.46	31.10	34.39	38.14
53	21.30	24.45	28. 5	31.30	35. 0	38 37
54	21.45	25. 2	28.24	31.50	35.21	38.58
55	22. 0	25.19	28.42	32. 9	35.42	39 19
56	22.14	25.35	28 59	32.28	36. 1	39.40
57	22.28	25.50	29.16	32.46	36.20	39.59
58	22.42	26. 5	29.32	33. 3	36.38	40.18
59	22.55	26.20	29.48	33.20	36.56	40.36
60	23. 8	26.34	30. 3	33.36	37 13	40.54

A Continuation of the Table.

H		II.X.		*		III. IX.
P	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
61	23.20	26.47	30.18	33.52	37.29	41.10
62	23.32	27. 2	30.34	34. 7	37.47	41.29
63	23.43	27.13	30.46	34.22	38. 0	41.42
64	23.54	27.25	30.59	34.25	38.15	41.57
65	24. 5	27.37	31.12	34.49	38.29	42.11
66	24.15	27.48	31.24	35. 2	38.42	42.25
67	24.24	27.59	31.35	35.14	38.55	42.39
68	24.34	28.10	31.46	35.26	39. 5	42.50
69	24.43	28.19	31.57	35.37	39.18	43. 2
70	24.52	28.29	32. 7	35.47	39.29	43.13
71	25. 0	28.38	32.17	35.58	39.39	43.23
72	25. 8	28.46	32.26	36. 7	39.50	43.33
73	25.16	28.54	32.34	36.16	39.59	43.43
74	25.22	29. 2	32.42	36.25	40. 8	43.52
75	25.28	29. 9	32.50	36.33	40.16	44. 0
76	25.34	29.15	32.57	36.40	40.24	44. 8
77	25.40	29.21	33. 4	36.47	40.31	44.15
78	25.45	29.27	33.10	36.53	40.37	44.22
79	25.50	29.33	33.16	36.59	40.43	44.28
80	25.54	29.37	33.21	37. 5	40.49	44.34
81	25.58	29.42	33.25	37.10	40.54	44.39
82	26. 2	29.45	33.29	37.14	40.58	44.44
83	26. 5	29.49	33.33	37.18	41. 2	44.47
84	26. 7	29.52	33.36	37.21	41. 5	44.51
85	26.10	29.54	33.39	37.24	41. 8	44.53
86	26.12	29.56	33.41	37.26	41.11	44.56
87	26.13	29.58	33.43	37.28	41.13	44.58
88	26.14	29.59	33.44	37.29	41.14	44.59
89	26.15	30. 0	33.45	37.30	41.15	45. 0
90	26.15	30. 0	33.45	37.30	41.15	45. 0

A Continuation of the Table.

H				IV. VIII.		
P	D. M	D. M.	D. M.	D. M.	D. M.	D. M.
1	1. 9	1.18	1.30	1.44	2. 2	2.25
2	2.17	2.36	2.59	3.28	4. 3	4.49
3	3.25	3.54	4.29	5.11	6. 3	7.12
4	4.33	5.11	5.58	6.53	8. 3	9.34
5	5.41	6.29	7.26	8.35	10. 1	14.53
6	6.48	7.45	8.54	10.16	11.58	14.10
7	7.54	9.41	10.20	11.55	13.53	16.24
8	9. 1	10.17	11.46	13.33	15.46	18.24
9	10. 7	11.31	13.11	15.10	17.36	20.42
10	11.12	12.45	14.34	16.44	19.24	22.45
11	12.16	13.58	15.56	18.17	21. 9	24 44
12	13.20	15.10	17.17	19.48	22.52	26.39
13	14.23	16.20	18.37	21.17	24.31	28.30
14	15.26	17.30	19.55	22.32	26. 8	30.17
15	16.26	18.39	21.10	24. 9	27.42	32. 0
16	17.27	19.46	22.25	25.31	29.12	33.38
17	18.26	20.52	23.38	26.51	30 40	35.13
18	19.25	21.56	24.49	28. 9	32. 4	36.43
19	20.22	23. 0	25.59	29.25	33.26	38.10
20	21.18	24. 1	27. 7	30.39	34.44	39 33
21	22.14	25. 2	28.16	31.50	36. 0	40.52
22	23. 8	26. 1	29.17	33. 3	37.13	42. 7
23	24. 1	26.59	30.19	34. 5	38.24	43 20
24	24.53	27.56	31.20	35.10	39.31	44.28
25	25.43	28.48	32.19	36.12	40.36	45.31
26	26.34	29.44	33.16	37.13	41.38	46.37
27	27.22	30.37	34.12	38.11	42.38	47.37
28	28.10	31.28	35. 5	39. 7	43.35	48.35
29	28.56	32.17	35.58	40. 1	44.33	49 29
30	29.41	33. 5	36.48	40.54	45.24	50 22

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A Continuation of the Table.

H		*		IV. VIII		*
P	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
31	30.25	33.52	37.37	41.44	46.15	51. 4
32	31. 9	34.58	38.25	42. 3	47. 3	51.59
33	31.51	35.22	39.11	43.20	47.50	52.45
34	32.31	36. 5	39.55	44. 5	48.35	53.28
35	33.11	36.47	40.39	44.49	49.19	54. 0
36	33.50	37.27	41.20	45.31	50. 0	54.50
37	34.27	38. 6	42. 0	46.11	50.40	55.28
38	35. 4	38.44	42.40	46.50	51.18	56. 4
39	35.40	39.21	43.17	47.28	51.55	56.39
40	36.14	39.57	43.53	48. 4	52.30	57.12
41	36.48	40.32	44.29	48.39	53. 4	55.44
42	37.21	41. 5.	44.58	49.13	53.37	58.14
43	37.51	41.38	45.35	49.46	54. 8	58.44
44	38.23	42. 9	46. 7	50.16	54.37	59.19
45	38.53	42.40	46.37	50.46	55. 7	59.38
46	39.22	43. 9	47. 7	51.15	55.34	60. 4
47	39.50	43.37	47.35	51.43	56. 0	60.29
48	40.17	44. 5	48. 2	52. 9	56.26	60.52
49	40.43	44.31	48.29	52.35	56.50	61.14
50	41. 8	44.57	48.54	53. 0	57.14	61.36
51	41.43	45.22	49.19	53.23	57.36	61.56
52	41.56	45.46	49.42	53.46	57.58	62.16
53	42.19	46. 9	50. 5	54. 8	58.18	62.35
54	42.42	46.30	50.27	54.29	58.38	62.53
55	43. 3	46.52	50.48	54.49	58.57	63.11
56	43.23	47.13	51. 8	55. 9	59.15	63.27
57	43.43	47.33	51.28	55.27	59.33	63.43
58	44. 2	47.52	51.46	55.15	59.49	63.58
59	44.21	48.10	52. 4	56. 2	60. 5	64.13
60	44.38	48.28	52.21	56.18	60.20	64.26

A Continuation of the Table.

H		*	.	IV VIII		*
P	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
61	44.55	48.44	52.37	56.34	60.35	64.39
62	45.14	49. 1	52.52	56.49	6049	64.52
63	45.27	49.16	53. 8	57. 3	61. 2	65. 4
64	45.42	49.31	53.20	57.17	61.15	65.15
65	45.56	49.45	53.36	57.30	61.27	65.26
66	46.10	49.58	53.49	57.42	61.38	65.37
67	46.23	50.11	54. 1	57.55	61.49	65.46
68	46.36	50.23	54.13	58. 5	61.59	65.55
69	46.47	50.35	54.24	58.16	62. 9	66. 4
70	46.59	50.46	54.35	58.26	62.19	66.13
71	47. 9	50.56	54.45	58.36	62.27	66.21
72	47.20	51. 6	54.55	58.44	62.36	66.28
73	47.29	51.16	55. 4	58.53	62.43	66.35
74	47.37	51.24	55.12	59. 1	62.50	66.41
75	47.48	51.32	55.20	59. 8	62.57	66.47
76	47.53	51.40	55.27	59.15	63. 4	66.53
77	48. 1	51.46	55.34	59.21	63. 9	66.58
78	48. 8	51.53	55.40	59.27	63.15	67. 3
79	48.13	51.59	55.45	59.32	63.20	67. 7
80	48.19	52. 4	55.51	59.37	63.24	67.11
81	48.24	52. 9	55.55	59.41	63.28	67.15
82	48.28	52.14	55.59	59.46	63.32	67.18
83	48.33	52.17	56. 3	59.49	63.35	67.21
84	48.36	52.21	56. 6	59.52	63.37	67.23
85	48.39	52.24	56. 9	59.54	63.40	67.25
86	48.41	52.26	56.11	59.56	63.42	67.27
87	48.43	52.28	56.13	59.58	63.43	67.28
88	48.44	52.29	56.14	59.59	63.44	67.29
89	48.45	52.30	56.15	60. 0	63.45	67.30
90	48.45	52.30	56.15	60. 0	63.45	67.30

A Continuation of the Table.

H		V.VII.		*		VI.
P	D.M.	D. M.	D. M.	D.M.	D.M.	D. M.
1	2.56	3.44	5. 1	7.33	14.55	90. 0
2	5.51	7.25	9.57	14.51	28. 2	90. 0
3	8.46	11. 3	14.44	21.41	38. 36	90. 0
4	11.37	14.36	19.20	27.55	46.47	90. 0
5	14.24	18. 1	23.30	33.30	53. 3	90. 0
6	17. 7	21.19	27.43	38.27	57.55	90. 0
7	19.45	24.28	31.30	42.47	61.44	90. [illegible]
8	22.17	27.27	34.59	46.35	64.47	90. 0
9	24.44	30.17	38.11	49.55	67.17	90. 0
10	27. 6	32.57	41. 7	52.50	69.19	90. 0
11	29.21	35.27	43.48	55.24	71. 2	90. 0
12	31.29	37.49	46.16	57.39	72.41	90. 0
13	33.32	40. 2	48.31	59.39	73.46	90. 0
14	35.29	42. 4	50.34	61.27	74.50	90. 0
15	37.19	44. [illegible]	52.27	63. 2	75.48	90. 0
16	39. 5	45.46	54.11	64.28	76.37	90. 0
17	40.44	47.29	55.46	65.45	77.22	90. 0
18	42.19	49. 4	57.14	66.55	78. 2	90. 0
19	43.48	50.32	58.35	67.59	78.37	90. 0
20	45.13	51.55	59.49	68.57	79. 9	90. 0
21	46.33	51.13	60.58	69.50	79.38	90. 0
22	47.49	54.26	62. 2	70.38	80.18	90. 0
23	49. 1	55.34	63. 1	71.23	80.29	90. 0
24	50. 9	56.37	63.56	72. 4	80.51	90. 0
25	51.14	57.37	64.48	72.42	81.11	90. 0
26	52.14	58.34	65.36	73.17	81.30	90. 0
27	53.13	59.27	66.20	73.50	81.47	90. 0
28	54. 8	60.17	67. 2	74.20	82. 3	90. 0
29	55. 0	61. 4	67.42	74.48	82.18	90. 0
30	55.50	61.49	68.18	75.15	82.32	90. 0

A Continuation of the Table.

H		V.VII.		*		VI.
P	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
31	56.37	62.31	68. 53	75.40	82.45	90. 0
32	57.21	63.10	69.25	76. 3	82 57	90. 0
33	58. 4	63.48	69.56	76.25	83. 8	90. 0
34	58.44	64.24	70.25	76.45	83.19	90. 0
35	59.23	64.58	70.53	77. 4	83.29	90. 0
36	60. 0	65.30	71.18	77.22	83.38	90. 0
37	60.34	66. 0	71.43	77.40	83.47	90. 0
38	61. 8	66.29	72. 7	77.56	83.55	90. 0
39	61.39	66.56	72.28	78.11	84. 3	90. 0
40	62.10	67.21	72.48	78.25	84.11	90. 0
41	62.39	67.47	73. 8	78.39	84.18	90. 0
42	63. 6	68.11	73.27	78.52	84.24	90. 0
43	63.32	68.33	73.44	79. 4	84.31	90. 0
44	63.57	68.54	74. 1	79.16	84.37	90. 0
45	64.21	69.15	74.17	79.28	84.42	90. 0
46	64.44	69.34	74.33	79.38	84.48	90. 0
47	65. 6	69.53	74.47	79.48	84.53	90. 0
48	65.27	70.10	75. 1	79.57	84.57	90. 0
49	65.47	70.27	75.14	80. 6	85. 2	90. 0
50	66. 6	70.43	75.27	80 15	85. 7	90. 0
51	66.24	70.59	75.39	80.23	85.11	90. 0
52	66.42	71.13	75.50	80.31	85.15	90. 0
53	66.58	71.27	76. 1	80.38	85.19	90. 0
54	67.14	71.41	76.11	80.45	85.22	90. 0
55	67.29	71.53	76.21	80.52	85.25	90. 0
56	67.44	72. 5	76.31	80.59	85.29	90. 0
57	67.58	72.17	76.39	81. 5	85.32	90. 0
58	68.11	72.28	76.48	81.11	85.35	90. 0
59	68.24	72.38	76.56	81.16	85.38	90. 0
60	68.36	72.48	77. 4	81.21	85.40	90. 0

A Continuation of the Table.

H		V.VII.		*		VI.
P	D.M.	D. M.	D. M.	D. M.	D.M.	D. M.
61	68.47	72.58	77.11	81.26	85.43	90. 0
62	68.58	73. 7	77.18	81.31	85.45	90. 0
63	69. 9	73.16	77.25	81.36	85.47	90. 0
64	69.19	73.24	77.31	81.40	85.50	90. 0
65	69.28	73.32	77.37	81.44	85.52	90. 0
66	69.37	73.39	77.43	81.48	85.54	90. 0
67	69.45	73.46	77.49	81.52	85.55	90. 0
68	69.53	73.53	77.54	81.55	85.57	90. 0
69	70. 1	73.59	77.58	81.58	85.59	90. 0
70	70. 8	74. 5	78. 3	82. 1	86. 0	90. 0
71	70.15	74.11	78. 7	82. 4	86. 2	90. 0
72	70.21	74.16	78.11	82. 7	86. 3	90. 0
73	70.27	74.21	78.15	82.10	86. 5	90. 0
74	70.33	74.25	78.18	82.12	86. 6	90. 0
75	70.38	74.30	78.22	82.14	86. 7	90. 0
76	70.43	74.34	78.25	82.16	86. 8	90. 0
77	70.47	74.37	78.28	82.18	86. 9	90. 0
78	70.52	74.41	78.30	82.20	86.10	90. 0
79	70.56	74.44	78.33	82.22	86.11	90. 0
80	70.59	74.47	78.35	82.23	86.12	90. 0
81	71. 2	74.49	78.37	82.24	86.12	90. 0
82	71. 5	74.52	78.39	82.26	86.13	90. 0
83	71. 7	74.53	78.40	82.27	86.13	90. 0
84	71. 9	74.55	78.42	82.28	86.14	90. 0
85	71.11	74.56	78.43	82.28	86.14	90. 0
86	71.12	74.57	78.44	82.29	86.14	90. 0
87	71.13	74.58	78.44	82.30	86.15	90. 0
88	71.14	74.59	78.45	82.30	86.15	90. 0
89	71.15	75. 0	78.45	82.30	86.15	90. 0
90	71.15	75. 0	78.45	82.30	86.15	90. 0

LEMMA XII.

PROBLEM.

How to find the Sun's Eaſtern or Weſtern Amplitude on a given Day for a given Latitude.

IMagine, as before, a Plain parallel to the Equator, and chooſe in it a Point, as A, for the Foot of the Style, or for One of the Poles of the World. Draw thro' the Point A the Indefinite Line AD, which you muſt take for the Meridian of the Place where you are, and draw to it, at the ſaid Point A, the Perpendicular AE, of what Length you pleaſe, which muſt be the Length of your Style, and then the Point E will be the Dividing Center of the Line AD, *by Lem.* 3. Then at the Dividing Center E, draw the Angle AEB, of the Height of the Pole above the Horizon, and the Angle AEC of the Diſtance of the Sun from the Pole, or of the Complement of the Sun's Declination on the given Day, which you will find *by Lem.* 10. Afterwards draw as before, thro' the Point B, FG, perpendicular to the line AD, which will be the Horizontal Line, as was ſhew'd in *Lem.* 11. and taking the Pole A for your Center, draw thro' the point C the Circle HGI, which will repreſent the Sun's Parallel, as we have alſo ſhew'd *in Lem.* 11. and which will upon the Horizontal FG, give G the point of the Sun's Riſing or Setting. Wherefore the Arch BG, will be the Complement of the Sun's Amplitude, whoſe Quantity will be known by ſetting off BE, at BD, and drawing the line DG, which will make the Angle BGD, with the Horizontal Line FG, whoſe Quantity being meaſur'd with a Protractor, or any other way, you will have the Amplitude requir'd.

Plate 5. *Fig.* 12.

DEMONSTRATION.

The Point D being the Dividing Center of the Horizontal Line BG, *by Lem.* 3. the Angle ADG will be the meaſure of the Part BG, or of the Complement of the Sun's Amplitude; and as the Angle GBD is the Complement of the Angle ADG, it follows that that Angle BGD is the Sun's Amplitude. *Q. E. I. & D.*

SCHOLIUM.

Inſtead of making Uſe of the Gnomonick Projection for the Solution of this Problem, you may eaſily make uſe of the Orthographical Projection of the Sphere, which you muſt

muſt make on the Plain of the Meridian of the place where you are, in the following Manner.

Plate 3. *Fig.* 16. Having drawn from the point A, as the true Eaſt Point, or Weſt Point, the Meridian of the Place, or the Circle BFCI, of what bigneſs you pleaſe, and having drawn at pleaſure the Diameter BC, which you muſt take for your Horizon, take the Two Arches BD, CE, each of the Complement of the Elevation of the Pole, and draw the Diameter DE which will repreſent the Equator. Mark alſo the Two Arches DF, EG, each being the Arch of the Sun's Declination, and draw the line FG which will repreſent the Sun's Parallel, and give upon the Horizon BC, H the Point of the Sun's Riſing or Setting, ſo that the part AH will be the Sun's Amplitude, whoſe Quantity will be known by meaſuring the Arch IK terminated by the Two lines AI, HK, which muſt be perpendicular to the line BC.

This Problem may alſo very eaſily and elegantly be ſolv'd by the Principles of the Stereographical Projection of the Sphere, which muſt be drawn thus, upon the Plain of the Horizon.

Having deſcrib'd from the point A taken for your Zenith or Nadir, the Horizon of the Place, or the Circle BFCI of what bigneſs you pleaſe, and having drawn at pleaſure the Diameter BC, which you muſt take for the Prime Vertical, draw to it the perpendicular *Radius* AI, which will Repreſent part of the Meridian. Then take the Arch BL of the Height of the Pole, and the Arch LM of the Sun's Declination, and draw to the *Radius* AL the Parallel MN, and to the *Radius* AI the Perpendicular NO, and the Arch BO will be the Amplitude requir'd.

This Amplitude may alſo be known another way and more exactly by Trigonometry in the Rectangular Spherical Triangle ABG, in which beſides the Right-angle B, the ſide AB which is the Elevation of the Pole, and the Hypotenuſe *Fig.* 12. AG of the Complement of the Declination of the Sun are known. Wherefore to find the other ſide BG, or the Complement of the Sun's Amplitude, this Analogy muſt be made:

As the Sine of the Complement of the Elevation of the Pole:
To the Whole Sine::
So the Sine of the Declination of the Sun:
To the Sine of the Sun' Amplitude.

According to this Method is the following Table calculated, which ſhews the Eaſtern Amplitudes for each Degree of the Pole's Elevation, from 30 Degrees Latitude to 60 Degrees; and for each Degree of Declination, from 1 Degree to 30 Degrees.

A Table of the Eaſtern Amplitudes, for different Latitudes.

P	31	32	33	34	35
D	D. M.	D. M.	D. M.	D. M.	D. M.
1	1.10	1.11	1.12	1.12	1.13
2	2.19	2.21	2.23	2.25	2.26
3	3.29	3 31	3.34	3.37	3.40
4	4.41	4.44	4.48	4.52	4.15
5	5.50	5.54	5.58	6. 2	6. 6
6	7. 1	7. 6	7.11	7.15	7.20
7	8.10	8.15	8.20	8.25	8.32
8	9.21	9.27	9.33	9.40	9.47
9	1031	10.38	10.45	10.53	11. 1
10	11.42	11.50	11.58	12. 6	12.17
11	12.51	12.59	13 .8	13.17	13.27
12	14. 2	14.12	14.22	14.32	14.42
13	15.11	15.22	15 33	15.44	15.56
14	16.24	16.35	16.46	16.58	17.11
15	17.34	17.46	17.59	18.11	18.25
16	18.45	18.58	19.11	19.25	19.40
17	19.57	20.10	20.24	20.39	20.55
18	21. 8	21.22	21.37	21.53	22.10
19	22.19	22.34	22.50	23. 7	23.25
20	23.31	23.47	24. 4	24.22	24.41
21	24.40	25. 1	25.19	25.37	25.57
22	25.55	26.13	26.32	26.52	27.13
23	27. 7	27.16	27.46	28. 7	28.29
24	28.20	28.39	29. 1	29.23	29.46
25	29.32	29.54	30.16	30.39	31. 4
26	30.46	31. 8	31.31	31.55	32.21
27	31.58	32.12	32.47	33.12	33.39
28	33.13	33.37	34. 3	34.30	34.58
29	34.27	34.52	35.19	35.47	36.18
30	35.41	36. 8	36.36	37. 5	37.38

A Continuation of the Table.

P	36	37	38	39	40
D	D. M.	D. M.	D. M.	D. M.	D. M.
1	1.14	1.15	1.16	1.17	1.18
2	2.28	2.30	2.32	2.34	2.37
3	3.43	3.45	3.48	3.51	3.55
4	4.57	5. 0	5. 4	5. 8	5.13
5	6.11	6.16	6.21	6.26	6.32
6	7.25	7.31	7.37	7.44	7.51
7	8.40	8.47	8.54	9. 1	9. 9
8	9.54	10. 2	10.10	10.19	10.28
9	11. 9	11.18	11.27	11.37	11.47
10	12.24	12.33	12.43	12.54	13. 6
11	13.38	13.49	14. 1	14.13	14.26
12	14 53	15. 5	15.38	15.31	15.45
13	16. 4	16.22	16.35	16.49	17. 5
14	17.24	17.38	17.53	18. 8	18.25
15	18.50	18.55	19.10	19.27	19.45
16	19.55	20.11	20.28	20.49	21. 5
17	21.11	21.28	21.46	22. 0	22.20
18	22.27	22.48	23. 5	23.26	23.48
19	23.44	24. 3	24.24	24.46	25. 9
20	25. 0	25.21	25.43	26. 6	26.31
21	26.18	26.39	27. 3	27.27	27.53
22	27.35	27.58	28.23	28.49	29.17
23	28.53	29.17	29.43	30.11	30.40
24	30.11	30. 7	31. 6	31.35	32. 4
25	31.30	31.57	32.27	32.58	33.29
26	32.48	33.17	33.48	34.24	34.54
27	34. 8	34.37	34.57	35.38	36.20
28	35.28	36. 0	36.33	37. 6	37.48
29	36.49	37.23	38. 1	38.38	39.16
30	38.10	38.46	39.26	40. 5	40.45

A Con-

A Continuation of the Table.

P	41	42	43	44	45
D	D. M.	D. M	D. M.	D. M.	D. M.
1	1.19	1.20	1.21	1.23	1.25
2	2.39	2.41	2.44	2.47	2.50
3	3.58	4. 2	4. 6	4.10	4.15
4	5.17	5.21	5.28	5.34	5.40
5	6.38	6.44	6.55	6.58	7. 5
6	7.58	8. 6	8.13	8.21	8.30
7	9.17	9.26	9.35	9.45	9.55
8	10.37	10.47	10.58	11. 9	11.21
9	11.57	12. 9	12.21	12.33	12.47
10	13.18	13.31	13.43	13.58	14.13
11	14.39	14.52	15. 7	15.22	15.39
12	16. 0	16.15	16.31	16.48	17. 6
13	17.20	17.37	17.55	18.14	18.33
14	18.42	19. 0	19.10	19.39	20. 0
15	20. 3	20.23	20.44	21. 5	21 28
16	21.25	21.46	22. 8	22.32	22.56
17	22.47	23.10	23.34	23.59	24.25
18	24.10	24.34	25. 0	25.26	25.54
19	25.33	25.59	26.26	26.54	27.24
20	26.57	27.24	27.53	28.23	28.56
21	28.20	28.49	29.20	29.54	30.27
22	29.45	30.16	30.49	31.24	31.59
23	31.10	31.43	32.17	32.54	33.32
24	32.40	33.16	33.53	34.30	35. 7
25	34. 6	34.43	35.23	36. 3	36.42
26	35.35	36.16	36.57	37.38	38.19
27	37. 3	37.46	38.30	39.13	39.57
28	38.35	39.20	40. 6	40.50	41.36
29	40. 4	40.52	41.40	42.28	43.17
30	41.36	42.27	43.18	44. 9	45. 0

A Continuation of the Table.

P	46	47	48	49	50
D	D. M.	D. M.	D. M.	D. M.	D. M.
1	1.26	1.27	1.29	1.31	1.33
2	2.53	2.56	2.59	3. 3	3. 7
3	4.19	4 24	4.29	4.34	4.40
4	5.46	5.52	5.59	6. 6	6.14
5	7.12	7.20	7.29	7.38	7.48
6	8.39	8.49	8.59	9. 9	9.21
7	10. 6	10.18	10.30	10.43	10.56
8	11.33	11.46	12. 0	12.15	12.31
9	13. 1	13. 16	13.31	13.47	14. 5
10	14.28	14.44	15. 2	15.21	15.40
11	15.57	16.15	16.34	16.54	17.19
12	17.23	17.45	18. 6	18.28	18.52
13	18.54	19.16	19.39	20. 3	20.29
14	20.23	20.46	21.12	21.28	22. 6
15	21.53	22.19	22.45	23.14	23.45
16	23.23	23.50	24.20	24.51	25.24
17	24.53	25.22	25.54	26.27	27. 3
18	26.25	26.50	27.30	28. 6	28.42
19	27.57	28.30	29. 6	29.45	30.20
20	30. 8	30.44	31.25	32. 9	32.55
21	31. 3	31.41	32.23	33. 6	33.53
22	32.38	33.19	34. 5	34.49	35.39
23	34.13	34.57	35.44	36.33	37.26
24	35.56	36.45	37.35	38.25	39.15
25	37.34	38.27	39.20	40.13	41. 6
26	39.13	40. 7	41. 2	41.57	42.52
27	40.52	41.57	42.57	43.57	44.57
28	42.39	43.43	44.45	45.51	46.55
29	44 25	44.33	46.41	47.49	48.58
30	46.12	46.24	48.36	49.50	51. 4

A Con-

A Continuation of the Table.

P D	51 D. M.	52 D. M.	53 D. M.	54 D. M.	55 D. M.
1	1.35	1.37	1.39	1.42	1.45
2	3.11	3.15	3.19	3.24	3.29
3	4.46	4.52	4.59	5. 6	5.14
4	6.22	6.30	6.39	6.49	6.59
5	7.58	8. 9	8.20	8.31	8.44
6	9.33	9.46	10. 0	10.14	10.30
7	11.10	11.25	11.41	11.58	12.16
8	12.47	13. 4	13.22	13.41	14. 3
9	14.24	14.44	15. 4	15.16	15.50
10	16. 1	16.23	16.46	17.11	17.37
11	17.39	18. 3	18.29	18.56	19.29
12	19.17	19.41	20.12	20.42	21.15
13	20.57	21.26	21.57	22.30	23. 5
14	22.36	23. 6	23.42	24.18	24.57
15	24.17	24.52	25.28	26. 7	26.45
16	25.58	26.35	27.15	27.58	28.43
17	27.40	28.21	29. 4	29.50	30.39
18	29.24	30. 8	30.54	31.43	32.38
19	31. 9	31.55	32.44	33.38	34.35
20	33.45	34.38	35.35	36.37	37.40
21	34.42	35.36	36.32	37.34	38.40
22	36.12	37.29	38.30	39.26	40.47
23	38.23	39.23	40.29	41.40	42.56
24	40.26	41.37	42.49	44. 0	45.10
25	42.22	43.28	44.55	46.10	47.28
26	44.13	45.34	46.56	48.18	49.41
27	46.26	47.54	49.23	50.51	52.20
28	48 37	50. 7	51.43	53.19	54.56
29	50.42	52.27	54.12	55.57	57.42
30	52.59	54.54	56.49	58.44	50.39

A Con-

A Continuation of the Table.

P	56	57	58	59	60
D	D. M.	D. M.	D. M.	D. M.	D. M
1	1.47	1.50	1.53	1.56	2. 0
2	3.35	3.41	3.47	3.53	4. 0
3	5.22	5.31	5.40	5.50	6. 1
4	7.10	7.22	7.34	7.47	8. 1
5	8.57	9.12	9.28	9.43	10. 2
6	10.46	11. 3	11.22	11.42	12. 4
7	12.35	12.56	13.18	13.41	14. 6
8	14.24	14.48	15.14	15.40	16.10
9	16.15	16.42	17.10	17.40	18.14
10	18. 6	18.36	19. 8	19.42	20.19
11	19.56	20.29	21. 6	21.46	22.26
12	21.50	22.27	23. 6	23.48	24.34
13	23.43	24.24	25. 7	25.54	26.44
14	25.38	26.22	27.10	28. 0	28.56
15	27.35	28.22	29.14	30.11	31.10
16	29.32	30.24	31.20	32.21	33.27
17	31.32	32.28	33.29	34.36	35.47
18	33.13	34.34	35.20	36.52	38.10
19	35.36	36.42	38. 2	39.12	40.37
20	37.43	38.54	40.12	41.36	43. 9
21	39.51	41.11	42.33	44. 5	45.46
22	42. 5	43.27	44.59	46.40	48.31
23	43.45	45.50	47.30	49.20	51.23
24	47. 1	48.52	50.43	52.34	54.26
25	49.30	51.32	53.35	55.38	57.42
26	51.59	54.[illegible]8	56.37	58.56	61.15
27	54.55	57.30	60. 5	62.40	65.14
28	57.55	60.54	63.54	66.53	69.53
29	61.20	64.53	68.35	72.13	75.51
30	63.26	66.39	70.39	76. 7	90. 0

LEMMA XIII.

PROBLEM.

How to Draw the Subſtylar Line *upon a Plain.*

THE *Subſtylar Line* is the Repreſentation of a Meridian, which goes thro' the Zenith of the Plain, and conſequently is perpendicular to the Horizon of the Plain, and to the Plain it ſelf; for which reaſon it is alſo call'd the *Meridian of the Plain*, upon which the Shadow of the End of the Style being come, it is Twelve a Clock to all thoſe whoſe Horizon is parallel to that Plain. As this Meridian is a great Circle, it is repreſented on the Plain by a Right-line, which will be thus found:

Plate 3. *Fig.* 15.

Chooſe upon the Plain a convenient Point, as A, for the Foot of the Style, and before you fix in it the Style AE, which muſt be rais'd at Right Angles upon the Plain, deſcribe from it ſeveral Concentrick Circles of what Bigneſs you will, which will repreſent *Almacanters* in reſpect of the Horizon of the Plain. Then a little after the Sun has begun to enlighten the Plain, and when the Shadow of the Style AE is not too Long, mark the End of that Shadow, when it touches one of the *Almacanters*, as B. It is plain that the Shadow of the End of the Style, will not be always in the Circumference of the ſame Circle, unleſs the Plain ſhou'd be parallel to the Equator; in which caſe it wou'd be inclin'd to the North, by the Degrees of the Complement of the Elevation of the Pole, and then the Vertical of the Plain wou'd be the Subſtylar Line requir'd: But the Shadow will become Shorter and Shorter, and not begin to grow Longer 'till the Sun has paſs'd the Meridian, whoſe Repreſentation is here requir'd. Stay then 'till the Shadow of the End of the Style touches the Circumference of the ſame Circles again, to mark a Second Point upon it, as C. Laſtly, Divide the Arch BC into Two equal Parts at D, and draw AD, which will be the Subſtylar Line requir'd.

DEMONSTRATION.

The Plain being conſider'd as an Horizon, the Foot of the Style A, will be the Zenith of that Horizon; the Lines, or rather the Shadows AC, AB, will repreſent the Sun's Verticals in reſpect of the ſaid Horizon, and the Circle BDC will repreſent the Circle of the Height of the Sun above the Plain;

Plate 3. *Fig.* 15.

becauſe in an Horizontal Plain, the Circles of Altitude are repreſented by Circles, becauſe in ſuch a caſe a Cone is cut by a Plain parallel to its Baſe. Now the Sun being One whole Day in the ſame Circle of Altitude, that is, in Two Points of the ſame Height above the Plain, as B, C; thoſe Two Points B, C, muſt alſo be equally diſtant from the Meridian of the Plain, and conſequently the Line AD, which Divides the Arch BC into Two equal Parts, muſt be the Repreſentation of that Meridian. *Q. E. I. & D.*

SCHOLIUM.

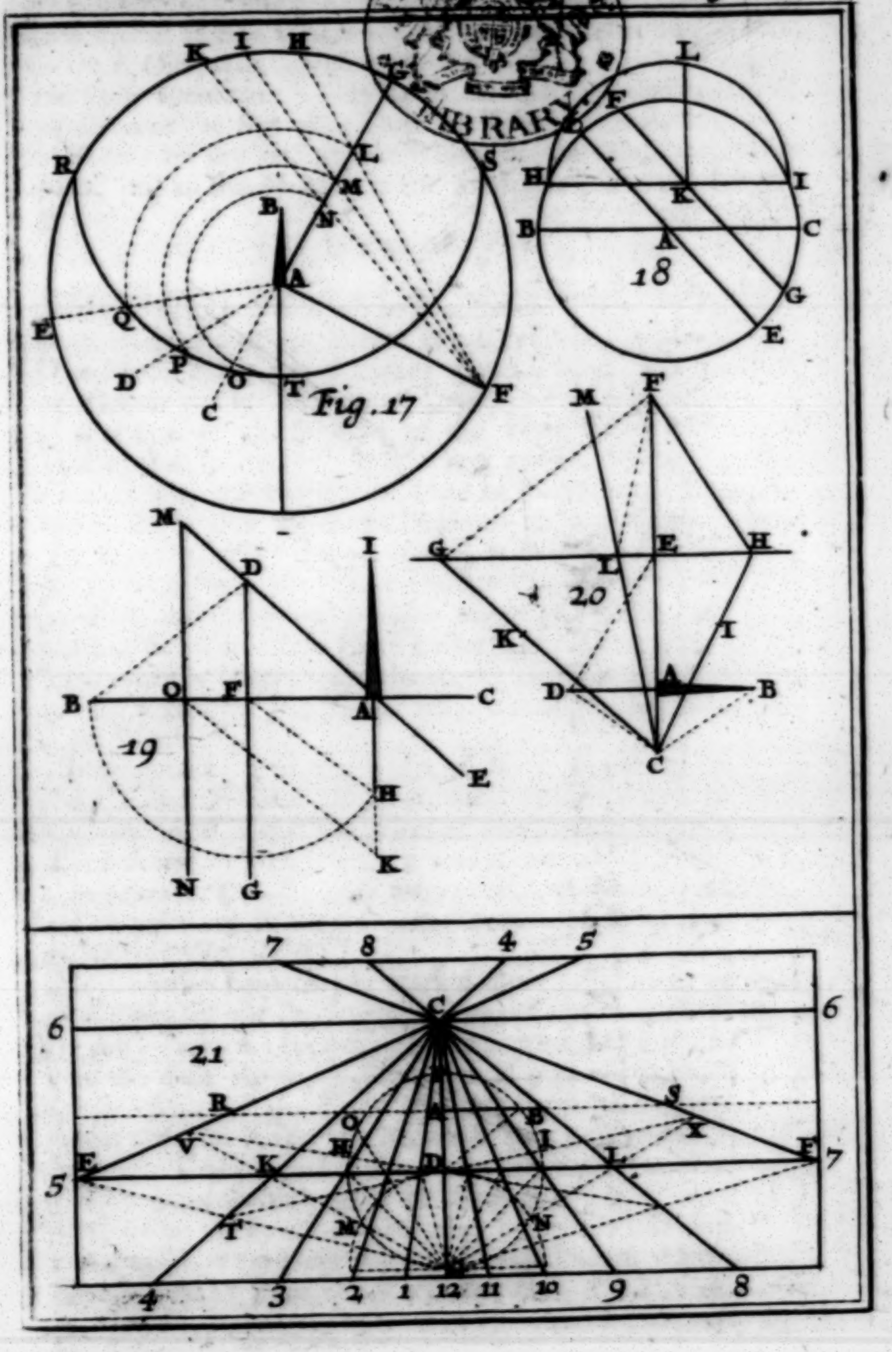
Plate 4. *Fig.* 17.

It may happen that the Sun will not ſhine all the while on the Plain, and then you cannot conveniently mark Two Points of Shadow in the ſame Circle; therefore inſtead of Two Limited Points, take Three Points at pleaſure; but the fartheſt you can from one another, as C, D, E, and for each Point of Shadow find the Height of the Sun upon the Plain, *by Lem.* 8. Afterwards deſcribe from the Foot of the Style A, the Circle EHF, with what Diſtance of the Compaſſes you pleaſe, and thro' the ſaid Point A, draw at pleaſure the Two Perpendicular *Radii* AF, AG, and take the Arch GH of the Height of the Sun above the Plain, (when the End of the ſhadow of the Style was at E) to draw the Line FH, which will upon AG give the Point L, thro' which, drawing a Circle from the Center A, you will upon AE have the Point Q. Take alſo the Arch GI of the Elevation of the Sun upon the Plain, (when the End of the ſhadow of the Style was at D) to draw the Line FI, which will upon AG give the Point M, thro' which, deſcribing a Circle from the Center A, you will upon AD have the Point P. Take alſo the Arch GK of the Elevation of the Sun upon the Plain, (when the End of the ſhadow of the Style was at C,) to draw the Line FK, which will upon AG, give the Point N, thro' which, deſcribing a Circle from the ſaid Cenrer A, you will upon AC have the Point O. Laſtly, deſcribe a Circle thro' the Three Points O, P, Q, as ROS, and thro' the Point T, the Middle of the Arch RTS, draw to the Foot of the Style A, the Right-line AT, which will be the Meridian of the Plain, or the Subſtylar Line requir'd.

DEMONSTRATION.

The Circle EGF being conſider'd as the Horizon of the Plain, its Center, or the Foot of the Style A, will be the Zenith; the Three Lines AC, AD, AE, will each repreſent the Vertical of the Sun, at the time that the Three Points of Shadow C, D, E, have been mark'd; the Three Points O, P, Q, will

Gnomonicks or Dialling Plate 4 Pag. 44



will each repreſent the Sun's Place on the Plain at that time; and the Circle R O S, will repreſent the Parallel or Diurnal Arch of the Sun, according to the Laws of the Stereographical Projection of the Sphere. Thus the Two Points R, S, will be the Points of the Setting and Riſing of the Sun, between which, juſt in the Middle, is the Subſtylar Line AT. *Plate* 4. *Fig.* 17. *Q. E. I. & D.*

SCHOLIUM.

This Subſtylar thus found upon an Horizontal Plain, will always be the Meridian of the Place. It will alſo be the Meridian of the Place, being found upon a Vertical Plain, when it is Perpendicular to the Horizontal Line; and then it will be known that one of the Surfaces of the Plain ſtands full *South*, and the other full *North*: But when upon a Vertical Plain, it makes with the Horizontal Line an Angle equal to the Elevation of the Pole above the Horizon, it will repreſent the Six Hours Circle; that is, it will be the Six a Clock Line, and it will ſhew that the Plain has one Surface towards the *Eaſt*, and the other towards the *Weſt*. Laſtly, it will alſo be the Meridian of the Place being found upon an Inclin'd Plain, when it is perpendicular to the Horizontal-line, and it will ſhew that the Plain Inclines towards one of the Poles of the World.

Aſtronomers make uſe of the Firſt of theſe Methods, to find the Meridian-line upon an Horizontal Plain, which is of uſe to 'em to find the Variation of the Needle touch'd with a Load-Stone, which formerly was ſcarce Diſcernable at *Paris*, and now is almoſt of ſix Degrees North-Weſt; that is, inſtead of pointing directly North, it declines from it about ſix Degrees Weſtward.

But this Meridian Line may be mark'd more eaſily upon an Horizontal Plain, by one ſingle Obſervation of the Shadow of the Style AE, (which I ſuppoſe Perpendicular to the Plain) *viz.* when the Sun Riſes or Sets; for if you make with the Shadow AC, of the Style AE (towards the Right Hand when the Sun Riſes, and towards the Left when it Sets) the Angle CAD, equal to the Complement of the Amplitude of the Sun, (which may be known by *Lem.* 12. if the Amplitude be the South-Amplitude) or to the ſaid Amplitude encreas'd by 90 Degrees, (if it be the North-Amplitude) the Line AD will be the Meridian requir'd. But this Meridian may be drawn yet another way upon an Horizontal Plain, and upon any other Plain whatever, at any Time, or any Hour of the Day, by means of One only Point of Shadow, as we ſhall Teach in *Lem.* 16 *Plate* 3. *Fig.* 15.

LEM-

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LEMMA XIV.

PROBLEM.

How to find the Sun's Vertical by means of one Point of Shadow mark'd upon a Plain.

THIS Point of Shadow will ſerve to know by *Lem* 9. the Sun's Height above the Horizon; by means of which, and its Declination, which may be known by *Lem.* 10. you may know how far the Vertical is diſtant from the Meridian, thus:

Plate 4. *Fig.* 18. Deſcribe at pleaſure from A for your Center, the Circle BFCE, which muſt be the Meridian of the Place, and the Colure of the Solſtices, and then the Center A will be the Eaſtern or Weſtern Equinoctial Point. Draw thro' the Center A the Diameter BC, which muſt be the Horizon; and having taken the Arch BD, or CE, of the Complement of the Elevation of the Pole above the Horizon, or equal to the Height of the Equator, draw the Diameter DE, which you muſt look upon as the Equator. Mark alſo the Arches DF, EG, (each being Arches of the Sun's Declination) above the Point D, if the Sun be above the Equator, as we ſuppoſe it here, and below the ſaid Point D, if the Sun be below the Equator, and draw FG, which being parallel to the Equator DE, will repreſent the Sun's Parallel. Laſtly, take the Arches BH, CI, (each of the Height of the Sun above the Horizon) and draw the Sun's *Almacanter* HI, which will upon the Parallel FG, give the Point K for the Sun's Place, according to the Laws of the Orthographical Projection of the Sphere: Wherefore, if from the Point K, you draw to the Line HI, the Perpendicular KL, which will be terminated at L, by the Semi-Circle HLI, deſcrib'd on the Line HI; the Arch HL, will give the Degrees of the Horizon, comprehended between the Meridian of the Place and the Vertical of the Sun, or the Diſtance from the Vertical of the Sun to the Meridian; that is, the Vertical requir'd.

SCHOLIUM.

As the Vertical of the Sun is of great Uſe, eſpecially to draw a Meridian Line upon a Plain, from which depends all the Exactneſs of a Dial; and ſince it is difficult to find out this Azimuth with a Ruler and Compaſſes, it will be better to find it out by Trigonometry, as we have taught at the End of our Trigonometry, **Book** 3. *Chap.* 4. *Queſt.* 9. and as we ſhall alſo Teach here.

Add

Add together the Three following things; the Diſtance from the Sun to the Pole which is rais'd above the Horizon; the Complement of the Height of the Sun; and the Complement of the Elevation of the Pole; and ſubtract ſeparately from half the Sum, the Complement of the Height of the Pole, and the Complement of the Height of the Sun, to have Two Differences; then make this Analogy:

As the Sine of the Complement of the Height of the Pole:
To the Sine of One of the Differences found ::
So the Sine of the other Difference:
To a Fourth Number.

As the Sine of the Complement of the Sun's Height:
To the Whole Sine ::
So the Fourth Number found:
To a Seventh Number.

And if that Seventh Number be Multiplied by the whole Sine, the Square Root of the Product will be the Sine of half of the Diſtance from the Vertical of the Sun to the North.

To find this Vertical at the Times of the Equinoxes, ſuch an Analogy as the following will ſuffice:

As the Whole Sine:
To the Tangent of the Height of the Pole ::
So the Tangent of the Height of the Sun:
To the Sine of the Complement of the Vertical of the Sun from the South.

LEMMA XV.

PROBLEM.

How to find the Declination of a Plain propos'd.

THE Plain here meant, muſt be either Erect or Inclin'd; becauſe an Horizontal Plain cannot be declining, ſince One of its Surfaces directly faces the Zenith, and the other the Nadir, its Situation is the ſame in reſpect to the Four Cardinal Points of the World: For by a *Declining Plain* we underſtand ſuch a Plain as does not directly face One of the Four Cardinal Points of the World; and by the *Declination of ſuch a Plain*, the Arch comprehended between the Prime Vertical, and the Vertical parallel to the Plain, when the Plain is Erect or Vertical, or the Arch of the Horizon comprehended between the Meridian of the Place, and the Vertical per-

perpendicular to the Plain, when the Plain is *Inclin'd*; that is, when it is neither Erect or Vertical, nor Horizontal: Or more generally, the Declination of a Plain is the Arch of the Horizon comprehended between the Meridian of the Place and the *Vertical of the Plain*; that is, the Vertical which is perpendicular to the Plain.

Plate 4. *Fig.* 19. Firſt, If the Plain be Erect, having by *Lem.* 7. drawn the Horizontal Line BC upon it, and by *Lem.* 13. the Subſtylar-line DE, take upon that Line DE, a Point at pleaſure, as D, for the Center of the Dial, from which you muſt let fall the Plumb-line DG, which muſt be your Meridian, without having any regard to the Length of the Style AI. At the Center D, draw FDB, the Angle of the Complement of the Height of the Pole, and from the Point F, thro' the Point B deſcribe an Arch, which upon AK, perpendicular to the Horizontal-line BC, will give the Point H, thro' which, if you draw to the Point F, the Line HF, the Angle AHF will be the Declination requir'd.

DEMONSTRATION.

If you take the Perpendicular AH for the Length of your Style, whoſe Foot is at A, the Point H will, *by Lem.* 3. be the Dividing Center of the Horizontal-line BC; and if the Angle AHF be taken for the Declination of the Plain, the Perpendicular DG will be the Meridian-line, whoſe Dividing Center is B, *by Lem.* 3. ſince the Line FB is equal to the Hypotenuſe FH: Wherefore, if the Pole or Center of the Dial is at D, thro' which the Subſtylar-line DE runs, the Angle FBD, which Meaſures the Part DF, muſt be equal to the Height of the Pole, and conſequently the Angle FDB, will be equal to the Complement of the Height of the Pole *Q. E.D.*

Secondly, If the Plain be Inclin'd, draw upon it the Horizontal Line, *by Lem.* 7. and having rais'd upon that Horizontal Line a Vertical Plain, find out the Declination of that Erect or Vertical Plain, as has juſt been Taught, and you will have the Declination of the Plain propos'd; which you may alſo have by means of a Magnetick Needle; but you muſt take notice of the Variation of the Needle, and take care that there be no Iron in, or near the Plain, which might draw the Needle out of its natural Poſition.

SCHOLIUM.

The Center D of the Dial has been taken above the Horizontal-line BC, becauſe we have ſuppos'd the Erect Plain to decline from the South; that is, its Surface to look obliquely towards the Lower Pole, it being certain that if the Sun was at the Lower Pole, and this Surface cou'd be

Plate 4. *Fig.* 19.

enlighten'd by the Sun, the Shadow of the Style wou'd tend upwards: For if the Plain ſhou'd decline from the North; that is, if the Surface of the Plain ſhou'd look obliquely towards the Upper Pole, as you may know when at Noon it is not enlighten'd by the Sun; the Center muſt have been taken below the Horizontal-line BC, becauſe if the Sun was in the Upper Pole, the Shadow of the Style wou'd tend downwards.

You need only do this when you wou'd have the true poſition of the Center; for when you wou'd know no more then the Declination of the Plain, you may take the Center either above or below the Horizontal-line. But this Center D, and the Meridian DG, is determin'd thus only in reſpect of a Style whoſe length is AH; and if you wou'd have it in reſpect of the Style AI, make the Perpendicular AK equal to the Length of that Style AI, and draw from the Point K to the line HF, the Parallel KO, which upon the Horizontal BC, will give the Point O, thro' which you muſt draw perpendicularly the line MN for your Meridian Line, and the point M where the Subſtylar Line meets it, will be the Center of the Dial in reſpect of the Style AI, *&c.*

LEMMA XVI.

PROBLEM.

How to draw the Meridian Line *upon a Plain.*

Plate 3. *Fig.* 15.

IF the Plain be Horizontal, the Subſtylar Line will be the Meridian requir'd, which may be mark'd upon that Plain, by means of the Sun's Eaſtern or Weſtern Amplitude, or otherwiſe, as has been taught by *Lemma* 13. or elſe by means of the Sun's Vertical, which having been found *by Lemma* 14. You muſt with the Line of Shadow AB, make the Angle BAD of the Sun's Vertical on the Right Hand, if the obſervation has been made in the After-noon; or with the Line of Shadow AC, the Angle CAD of the Sun's Vertical on the Left, if the Obſervation has been made in the Fore-noon, and the Line AD will be the Meridian.

Plate 2. *Fig.* 8.

But if the Plain be Vertical, the Subſtylar Line will alſo be the Meridian requir'd, when it happens to be perpendicular to the Horizontal-line; and when it makes with the ſaid Horizontal-line an Acute Angle greater than the Elevation of the Pole above the Horizon, (for it cannot make a leſs) the Plain will be Declining, whoſe Declination having been found *by Lemma* 15. you muſt at the point F, which is the Dividing Center of the Horizontal-line DE, (which before muſt have been drawn upon the Plain, *by*

Plate 2. *Fig.* 8. *Lemma* 7.) draw the Angle CFG of the Declination of the Plain, on the Right or on the Left as the Plain ſhall decline Eaſtward or Weſt-ward, and thro' the point G draw the Perpendicular AB, which will be the Meridian Line.

This Meridian may alſo be drawn upon a Vertical-plain without knowing either its Declination or the Sun's Vertical, by means of Two Points of Shadow mark'd the ſame Day upon the ſaid Plain as H, I, when the Sun has the ſame Height above the Horizon, which you may know by obſerving when the end of the Shadow of a Style fix'd perpendicularly to any Horizontal Plain, falls before Noon, and in the After-Noon upon the Circumference of the ſame Circle whoſe Center is the Foot of the Style, as if you wou'd mark upon it the Subſtylar Line: For if thro' thoſe Two Points H, I, you draw to the Horizontal-line DE, the Two Perpendiculars DH, KI, which will repreſent Two Verticals of the Sun equally diſtant from the Meridian, and after having drawn from F the Dividing Center of the Horizontal-line DE, the lines FD, FK, you divide the Angle DFK into Two equal parts by the line FG, you will upon the Horizontal-line DE have the South Point G, thro' which you may draw perpendicularly the Meridian AB.

Laſtly, if the Plain be Inclin'd, you muſt draw upon it the Meridian Line by means of its Declination, thus; Having *by Lemma* 7. found the Zenith or Nadir C, and the Horizontal-line GH, ſet off upon CD the Vertical of the Plain the Hypotenuſe EF, from F to D, which *Lemma* 3 will be the Dividing Center of the Horizontal GH, and at the point D draw the Angle of Inclination FDI, on the Right or on the Left according as the Declination is, to have upon the Horizontal GH, the South Point at I, thro' which and the Zenith C, you muſt draw the Right-line CK, which will be the Meridian requir'd.

The Line DI is call'd the *Line of Declination*, which may in this and in the Vertical Plain be parallel to the Horizontal, *viz.* when the Declination of the Plain is exactly of 90 Degrees; and then you muſt draw the Meridian parallel to the Horizontal-line in the Inclin'd Plain, drawing it always thro' the Zenith or Nadir C

Fig. 11. If you wou'd make uſe of the Sun's Vertical, having drawn thro' the Point of Shadow I, and the Zenith or Nadir C, the Vertical CK, and having found as we ſaid before, the Dividing Center D of the Horizontal GH, draw the line DK, and with it make the Angle KDH of the Sun's Vertical, (on the Right or Left, according as the Obſervation has been made before or after Noon,) to have upon the Horizontal-line the South Point at H, thro' which and the Zenith C, you will draw the Meridian CH.

If

If you wou'd make uſe of Two points of Shadow mark'd the ſame Day upon the Inclin'd Plain, at theſe times before, and after Noon that the Sun is at the ſame Height above the Horizon, as I,K; draw from the Zenith or Nadir C, thro' the points I,K, the Verticals CH, CG, which will be equally diſtant from the Meridian, and the point of that Meridian will conſequently be in the middle of the Arch of the Horizon GH; wherefore to find that middle Point, you muſt draw from the Dividing Center F, thro' the points G,H, where the Horizontal is cut by the Two Verticals of the Sun, the Rays FG, TH, and the Angle GFH muſt be divided into Two equal parts, by the line FL, which will upon the Horizontal GH, give the South Point at L, thro' which, and thro' the Zenith, or Nadir, you muſt draw the Meridian-line CM. *Plate 4. Fig. 20.*

CHAPTER II.

Of Horizontal Dials.

WE call *Dial* in General, an Inſtrument which ſhews the Hours by means of the Sun, Moon, or Stars, whether it be by Rays ſent directly, or by Reflection, or by Refraction: And when we ſay only Dial, it muſt be underſtood of a Sun-Dial, which is call'd a *Horizontal Dial*, when it is drawn upon an Horizontal Plain, whoſe different Conſtructions we ſhall here teach according to the ſeveral Caſes which may happen.

PROBLEM I.

How to draw an Horizontal Dial, *beginning at the Foot of the Style, in the* Oblique Sphere.

HAving determin'd the Point A for the Foot of the Style, draw at pleaſure thro' that Point A, the Meridian CG, if the Plain is moveable, otherwiſe you muſt draw this Meridian CG as has been taught *in Lem.* 16. of Chap. 1. Draw thro' the Foot of the Style A, the Line AB equal to the Length of the Style, and perpendicular to the Meridian CG, to have at B the Dividing Center of the Meridian. At that Point B towards the South, draw the Angle ABC of the Complement of the Elevation of the Pole, to have the Pole, or Center of the Dial at the Point C. Draw on the other ſide, that is, toward the South, the Angle ABD of the Height of the Pole, to have at D the Point of the Equator, thro' which *Fig. 21.*

Plate 4. *Fig.* 21. you muſt draw to the Meridian CG, the Perpendicular EF, which will be the *Equinoctial Line*, or Repreſentation of the Equator. Set off the Line DB, which is call'd the *Radius of the Equator*, upon the Meridian from D to G, which will be the Dividing Center of the Equinoctial EF, and which is call'd the *Center of the Equator*. Laſtly, deſcribe from that Center of the Equator G, the Circle MDN, and divide it into Parts of 15 Degrees, beginning at the Point D of the Meridian, to draw from the Center G thro' the Points of Diviſion *Radii*, which will upon the Equinoctial EF give the Horary, or Hour-points, thro' which you muſt draw from the Center of the Dial C, the Hour-lines, which you muſt produce beyond the Center C, to have the Lines of thoſe Hours at which the Sun riſes or ſets before Six. As for the Six a Clock Line, whoſe Point cannot be found upon the Equinoctial-line, it muſt be be drawn perpendicular to the Meridian, or parallel to the Equinoctial.

The Style rais'd at Right Angles on the Point A, of the Length AB, will ſhew the Hours when the Sun Shines, by the End of its Shadow, when the Center of the Dial C is directly turn'd towards the South, which may be eaſily ſet ſo by means of a Magnetick Needle, when the Plain is moveable. But inſtead of a Right, or Erect Style, it is more convenient to make uſe of an *Axis*, as BC, raiſing perpendicularly upon AC the Triangle ABC, which we ſhall call the *Triangular Style*, which may be as long as you will, if the *Axis* BC makes an Angle with the Plain, equal to the Height of the Pole, becauſe its Shadow by covering the Hour-lines will more plainly ſhew the Hours than the end of the Style, whoſe Shadow goes eaſily out of the Plain, when the Sun is not very high above the Horizon.

SCHOLIUM.

If the Length of the Style AB had been taken ſomewhat greater, you cou'd not have mark'd upon the Equinoctial EF, the Two laſt Points EF of 5 and Seven a Clock, becauſe they wou'd have fall'n out of the Plain, and therefore the 5 and Seven a Clock Lines cou'd not have been drawn. In ſuch a Caſe you may find Points for the 5th and 7th Hour upon another Line nearer the Center of the Dial C, and parallel to the Equinoctial EF, as upon the Line RS, ſetting off the diſtance of the Points of the Hours 3 and 9 taken upon that Line, from the Point 4 to R, which will be the Point of the 5th Hour, and likewiſe from the Point of 8 upon the ſame Line to S, which will be the Point of the 7th Hour.

You may after another Manner find the ſaid Points of the 5th and 7th Hour, thus; Take at pleaſure One of the Hour-

lines

lines already drawn, *for Example*, the Four a Clock Line, and mark the Point T where it cuts the Ray GE of the 5th Hour, and draw thro' that Point T, and the Interſection D of the Meridian and Equinoctial, the Indefinite-line TX, which will be terminated at X, by the Interſection of the Ray GL, of the 8th Hour, as far diſtant from the Meridian as the Four a Clock Line which you have taken, and that Point of Interſection X will be the Seven a Clock Point, thro' which may be drawn from the Center of the Dial C, the Line of the 7th Hour, and after the ſame Manner That of the 5th Hour.

Plate 4. Fig. 21.

DEMONSTRATION.

For the Demonſtration of this Operation, which is not common, produce GK, the Four a Clock Ray, till it meets CE the Five a Clock Line in a Point, as V, and draw the Lines DV, DX. I ſay that the Line CK is as far from the Meridian CG, as the Line CE, which is the 5 of the Clock Line; that is, the Angle GCX is equal to the Angle GCV.

For in the Triangle CEG, whoſe Baſe is CG, and Perpendicular ED, which falls within it, the Angle EDT is equal to the Angle EDV, *by Lem.* 4 *Chap.* 1. and conſequently to the Angle FDX; and becauſe the Obtuſe Angle DKV is alſo equal to the Obtuſe Angle DXL, by reaſon of their being made by Rays equally diſtant from the Meridian, which cauſes alſo the Two Lines DK, DL, to be equal, it follows, *by* 26. 1. that the Two Triangles DKV, DLX, are equal, and that the Side DV is equal to the Side DX: And becauſe the Side CD is common to each of the Triangles CDV, CDX, and the Angle CDV is equal to the Angle CDF, becauſe they are the Complements of the Two equal Angles EDV, FDX, it follows, *by* 4. 1. that theſe Two Triangles CDV, CDX, are equal to one another, and that the Angle DCX is equal to the Angle DCV. *Q. E. D.*

The Five and Seven a Clock Lines may be alſo drawn another way, as we ſhall teach in the next Problem, and the Equinoctial Line EF may be alſo divided into Hours, with only One opening of the Compaſſes, thus:

Deſcribe from the Center D, thro' the Point B, the Circle GHPI, which will go thro' the Center of the Equator G, and give upon the Equinoctial-line EF, the Two Points HI, of the 3d and 9th Hour, and ſet off the ſame diſtance upon the Circumference of that Circle to G, on either ſide to M, and N, and from theſe Two Points M, N, upon the Equinoctial EF, to the Points K, L, which will be the 4 and 8 a Clock Points, ſet off the ſame diſtance Twice, from K to E, which will be the 5 of the Clock Point, and if you do as much on

Plate 4. *Fig.* 21. the other ſide, from the Point K, you will have the Eleven a Clock point. Likewiſe ſet off the ſame diſtance Twice, from L to F, which will be the Seven a Clock Point; and if you do as much on the other ſide, from the ſaid Point L, you will have the One a Clock Line. Laſtly, ſet off the ſame diſtance from M to P upon the Circumference of the ſame Circle at O, thro' which drawing from the Point G the Ray GO, you will have on the Equinoctial the Two a Clock Point, and if you do the ſame thing on the other ſide, you will upon the ſaid Equinoctial have the Ten a Clock Point.

This Method is eaſier in the Practice than the Firſt, and conſequently leſs liable to Errors; becauſe in the Firſt Method if you happen to be ever ſo little out, in dividing the Circle MDN, or in drawing the *Radii* from the Center G, thro' the point of Diviſion of the Circle MDN, (even tho' it ſhou'd be very exactly divided) you will err in thoſe Horary-points, which are the fartheſt diſtant from the point D of the Meridian. But we will give you a Third Method for dividing the Equinoctial into Hours, which is yet to be preferr'd to the foregoing, becauſe it is done without deſcribing a Circle, as you will ſee.

Set off GD the diſtance of the Center of the Equator, or the Length of the *Radius* of the Equator DB, on either ſide upon the Equinoctial EF, from D to the point H,I, which will be the Points of Three and Nine a Clock. Set off the diſtance HI of thoſe Two Points, on either ſide upon the ſame Equinoctial EF, from the Center of the Equator G, to the points K,L, which will be the Points of the 4th and 8th Hour, and from K to E, which will be the Point of the 5th Hour, and alſo on the other ſide to have the Point of the 11th Hour: And likewiſe from L to F, which will be the Seven a Clock Point; and alſo on the other ſide for the One a Clock Point. The 4 and the 10 of the Clock Points are the only Points wanting, which will be found by dividing the diſtance KL, from the 4th to the 8th Hour, into three equal parts.

If beſides the Hour-points, you wou'd have the Half-hour-points, the Circle MDN muſt be divided into Twice as many parts, and into Four times as many, if you wou'd have the Quarters, *&c.* But theſe Points may be found by a ſhorter Method, thus: Having ſet One of the Legs of your Compaſſes on theſe Horary-points mark'd upon the Equinoctial-line, whoſe Number is odd, *viz.* on the Points of 5, 3, 1, 11, 9, and Seven a Clock, extend the other Leg of the Compaſſes as far as G the Center of the Equator, to have Diſtances which being ſet off on either ſide upon the Equinoctial, from the ſame Hour-points, you will have the Half-hours, by means of which you may likewiſe find the Quarters;

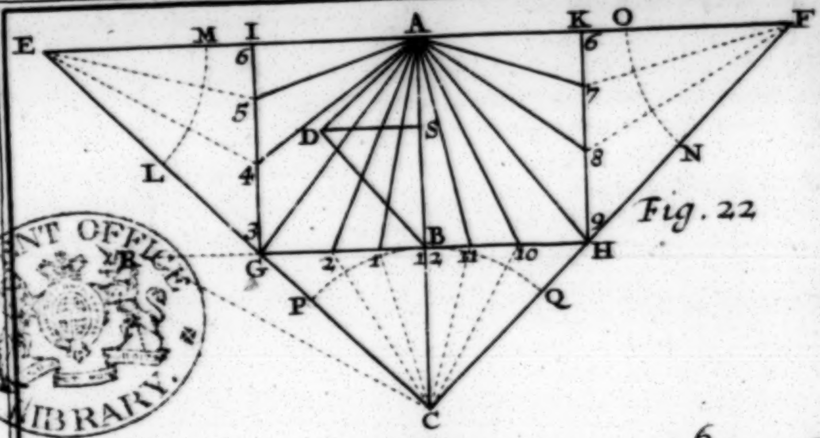
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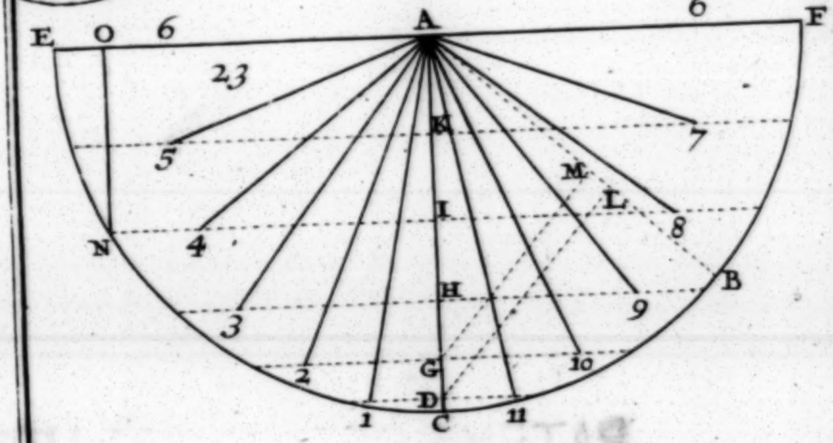
Fig. 22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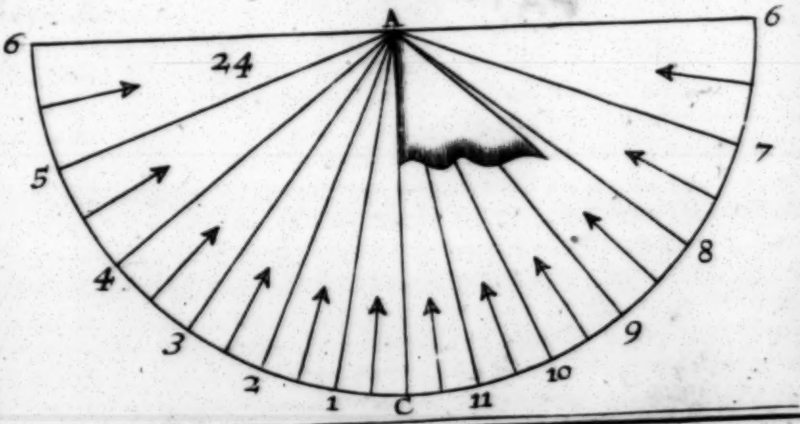
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ters, *&c.* The Demonſtration of theſe Operations are ſo eaſy that I need not give it.

PROBLEM II.

How to deſcribe an Horizontal Dial, *beginning by the Center of the Dial in the* Oblique Sphere.

Plate 5. *Fig.* 22.

IF the Center of the Dial be at the point A, draw from that point A the Two Perpendiculars AC, EF, of which AC being the Meridian, EF muſt be the Six of the Clock Line. Make at the point A, with the Meridian AC, the Angle CAD of the Height of the Pole above the Horizon of the Place where you are, (as at *Paris* of 49 Degrees, not minding the Minutes which ſignify little,) by means of the Line AD, which will repreſent the *Axis* of the World, which may be either Longer or Shorter according to the Length and Breadth of the Plain. Draw to that *Axis* AD thro' its end D, the Perpendicular DB, which will be the *Radius* of the Equator, and will give upon the Meridian AC, the point B of the Equator, thro' which you muſt draw to the Meridian AC the Perpendicular GH, which will be the Equinoctial-line. Draw alſo thro' the ſaid Point D, to the Meridian AC, the Perpendicular DS, which will determine the Length of the Style, and its Foot, at the point S. Laſtly, ſet off the *Radius* of the Equator BD on BC, to have at C the Dividing Center of the Equinoctial GH, which may be divided into Hours, as has been taught *in Probl.* 1. But as the laſt Hour-points will not be ſo exact, it will be better to end the Dial thus:

Having upon the Six of the Clock Line EF, taken the Two lines AE, AF, each equal to the part AC of the Meridian, draw the lines CE, CF, which will give upon the Equinoctial the Two points G, H, of the 3d and 9th Hour, from whence you muſt draw the Two lines GI, HK, perpendicular to the Equinoctial-line GH, or parallel to the Meridian AC. Then deſcribe from the Points C, E, F, the Arches PQ, LM, NO, to divide each of 'em into parts of 15 Degrees, and to end the reſt as you ſee in the Figure.

DEMONSTRATION.

If an Hour-line be produc'd, (*for Example,* the Four of the Clock Line) till it meets the Equinoctial at a Point, as R, thro' which you draw to the Center of the Equator C, the Ray CR, that Ray CR being the 4 a Clock Ray, will make the Angle ACR of 60 Degrees: But to demonſtrate that the Angle ACR is juſt of 60 Degrees, and conclude thence that the line AR is truly the 4 a Clock Line, it will ſuffice to demonſtrate that the Rectangular Triangle CBR is Similar *Plate* 5. to the Rectangular Triangle EI4, where the Angle E4I is *Fig.* 22. of 60 Degrees, which is evident in the Similar Triangles ABR, AI4, where you ſee *by* 4. 6. that the Four Lines AB : BR: : I4 : IA, or the Four IE : BR: : I4 : BC, are proportional, *&c.*

SCHOLIUM.

As this Operation requires the Plain to be pretty Large, that you may mark upon the Six a Clock Line EF, the Two points E, F, I ſhall add another Method to deſcribe more eaſily a Horizontal Dial up on a little Plain, without giving its Demonſtration, becauſe it depends on the *Fig.* 23. Principles of the *Orthographical Projection of the Sphere*, of which we have given no particular Treatiſe.

Deſcribe from the Center A, with what diſtance you pleaſe, the Semi-Circle ECF, which will be divided into Two equal parts at C by the Meridian AC. Divide each Quarter of the Circle CE, CF, into Six equal parts, and draw thro' the Points of Diviſion, Lines parallel to the Six a Clock Line EF, which will be divided at Right-angles, and into Two equal parts by the Meridian AC, at the Points D, G, H, I, K. Take the Arch CB of the Degrees of the Pole's Elevation above the Horizon, and draw the *Axis* AB, by means of which, you muſt mark the Hour-points upon the foregoing Parallels, ſetting off the diſtance from the Firſt point K to the *Axis* AB, upon the Firſt oppoſite Parallel, from D on either ſide to the points 1, 11, and likewiſe the diſtance from the Second point I to the ſaid *Axis* AB, upon the Second oppoſite Parallel, on either ſide from G to the points 2, 10, and ſo on.

Theſe diſtances will be found by drawing from the points D, G, H, I, K, as many Lines perpendicular to the *Axis* AB, as DL, GM, *&c.* or more eaſily by deſcribing from the ſaid points D, G, H, I, K, Arches which touch the *Axis* AB, which may be done eaſily at Sight, without any ſenſible Error from the true Length of thoſe Perpendiculars.

DEMONSTRATION.

Tho' we cannot here give a Geometrical Demonſtration of this Operation, becauſe, as we have ſaid before, it depends upon the Principles of the *Orthographical Projection of the Sphere*: Yet we may be ſure that it is right, by ſhewing, that the ſame Analogy, which is found by the Principles of Gnomonicks, to know the Horary-angles, is alſo found by this Method, thus:

To find, *for Example*, the Horary Angle of the Four a Clock Line, or the Angle CA4, which the Four a Clock Line

A4 makes with the Meridian AC, you muſt conſider that the Line AG is the Sine of the Horary Diſtance, in reſpect of the Whole Sine AC, becauſe it is equal to the Sine NI of the Horary Diſtance, CN: And becauſe the Line AI is equal to the Sine of the Complement of the ſaid Horary Diſtance, in reſpect of the Whole Sine AC, becauſe it is equal to the Whole Sine NO of the Arch EN, which is the Complement of the Horary Diſtance CN. *Plate* 5. *Fig.* 23.

Becauſe in the Triangle AMG, rectangular at M, the Sine of the Right-angle M, or the Whole Sine: is to its oppoſite ſide AG:: as the Whole Sine of GAM the Angle of the Elevation of the Pole above the Horizon: is to the Sine of its oppoſite ſide GM, if *d* be put for AG, or for the Sine of the Horary Diſtance, *s* for the Sine of the Angle GAM, or for the Sine of the Height of the Pole, and *r* for the Sine of the Right-angle M, or for the *Radius*, you will have $\frac{ds}{r}$ for the Line GM, or for the Line I4. its equal.

Becauſe in the Triangle AI4 Rectangular at I, the Side AI: is to the Side I4:: as the *Radius*: is to the Tangent of the Horary Angle AI4; if you put *c* for AI, or for the Sine of the Complement of the Horary Diſtance, you will have $\frac{ds}{c}$ for the Tangent of the Horary Angle IA4, and by reducing the Fraction $\frac{ds}{c}$ into Proportion, you will have this Analogy:

As the Sine of the Complement of the Horary Diſtance:
To the Sine of the Horary Diſtance::
So the Sine of the Elevation of the Pole:
To the Tangent of the Horary Angle.

And if inſtead of the Two firſt Terms, (*viz.* the Sine of the Complement of the Horary Diſtance, and the Sine of the Horary Diſtance) you put the Whole Sine, and the Tangent of the Horary Diſtance, you will have this other Analogy:

As the Whole Sine:
To the Tangent of the Horary Diſtance::
So the Sine of the Height of the Pole:
To the Tangent of the Horary Angle.

which is the ſame as that which is taken from the Principles of Gnomonicks, as you will ſee.

Since by the general Conſtruction of the Horizontal Dial, the Angle SDB is equal to the Elevation of the Pole, and the Angle SDA is equal to the Complement of the ſaid Height

Plate 5. *Fig.* 22. Height of the Pole above the Horizon, if you take the Length of the Style DS for the *Radius*, or for the Whole Sine, which we will call r, the Line BS will be the Tangent of the Elevation of the Pole, which we will call t, the Line BD will be the Secant of the Elevation of the Pole, which we will call s, and the Line AS will be the Tangent of the Complement of the Elevation of the Pole, which we will call c.

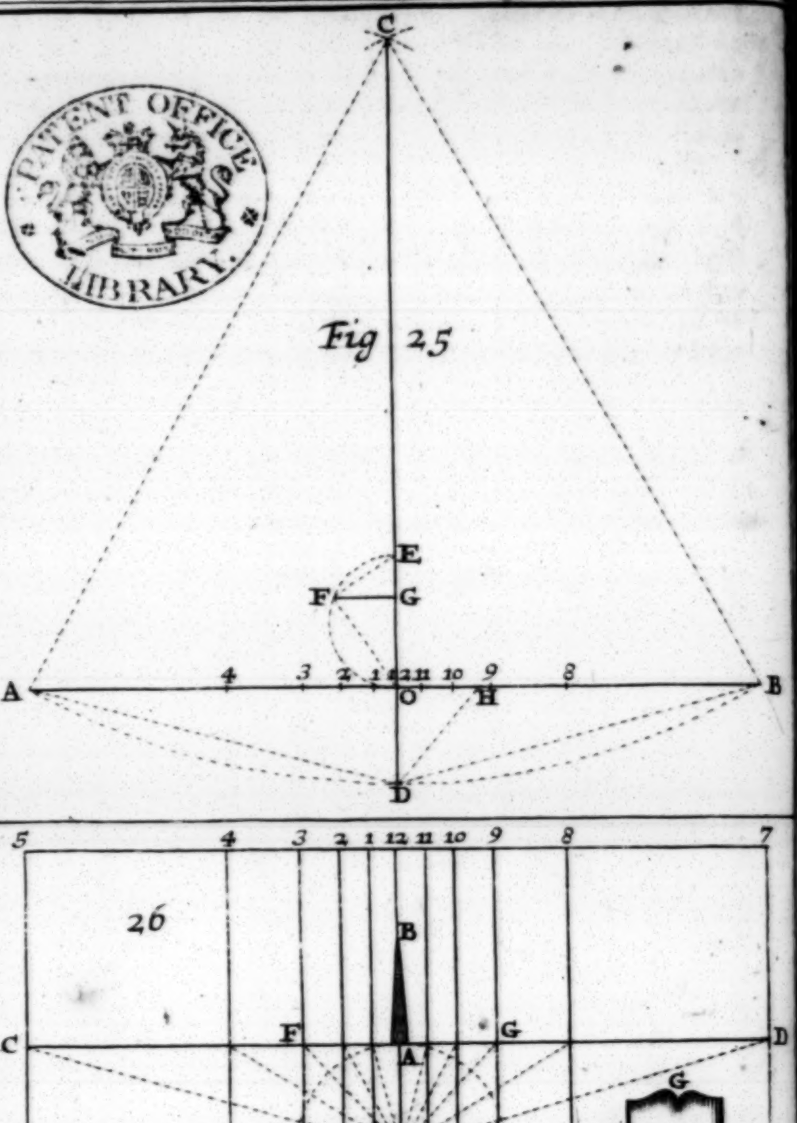
This Preparation being made, you muſt conſider, that ſince we have ſuppos'd $AS=c$, and $BS=t$, you will have $AB=c+t$: and becauſe BD has been ſuppos'd equal to s, its equal BC will alſo be equal to s, which being taken for the Whole Sine, the Line DR will be the Tangent of the Horary Diſtance, or of the Angle BCR, which you find in the Table for the Whole Sine r. And if you call it d, you will find $BR=\frac{ds}{r}$, becauſe in the Rectangular Triangle CBR, the Whole Sine: is to the Tangent of the Angle BCR:: as the Side BC: is to the Side BR.

Becauſe in the Rectangular Triangle ABR, the Side AB, or $c+t$: is to the Side BR or $\frac{ds}{r}$:: as the Whole Sine, or r: is to the Tangent of the Horary Angle BAR, you will have $\frac{ds}{c+t}$ for the Tangent: And becauſe in the Rectangular Triangle ADB, the Angle BAD is equal to the Elevation of the Pole, if you call a its Sine, which is found in the Tables, for the ſaid Whole Sine r, you will find AB, or $c+t=\frac{rs}{a}$. Wherefore if inſtead of $c+t$, you put $\frac{rs}{a}$, inſtead of $\frac{ds}{c+t}$, which we have found for the Tangent of the Horary Arch BAR, we ſhall have $\frac{ad}{r}$, whence is deduc'd this Analogy:

As the Whole Sine:
To the Tangent of the Horary Diſtance ::
So the Sine of the Height of the Pole :
To the Tangent of the Horary Angle.

which is the ſame as the foregoing, and which we have drawn from Spherical Trigonometry, in the Conſtruction of the Horary Arches, which are the Meaſure of the Horary Angles, from whence may be drawn a ſhorter Method than the foregoing, for the Deſcription of a Horizontal Dial, always beginning by its Center: For if from the Center of the Dial A, you deſcribe at pleaſure the Circumference of a Circle, which will repreſent the Horizon, to take upon it on either

Fig 25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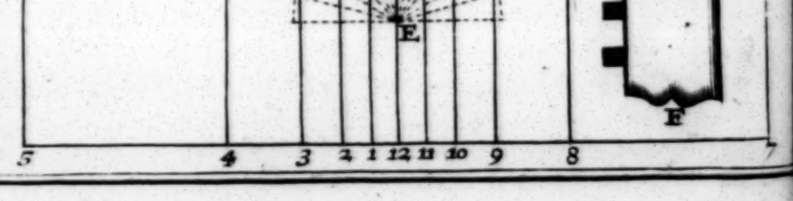
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either ſide from the South Point C, the Degrees and Minutes of the Horary Arches, which is found in the Table of Page 23, over-againſt the Degrees of the Height of the Pole above the Horizon of the Place where you are; and from the Points mark'd upon that Circumference you draw to the Center A, the Hour-lines, you will have as exact a Dial as can be drawn. *Plate* 5. *Fig.* 22.

PROBLEM III.

How to draw an Horizontal Dial *beginning at the Points of* 5 *and* 7 *a Clock, mark'd upon the* Equinoctial Line.

LEAST it ſhou'd happen, that all the Hour-Points cannot be mark'd upon the Equinoctial-line, you may begin the Dial by the ſaid Equinoctial, taking its Two Ends for the Points of 5 and 7 a Clock, after this manner:

Having determin'd the length AB of the Equinoctial at pleaſure, and taken its end A for the Five a Clock Point, and the other end B for the Point of 7, divide that Equinoctial AB into Two equal Parts at the Point O, which will be the South Point. Deſcribe upon the ſaid Equinoctial AB, the Equilateral Triangle ABC, and draw the Meridian CO, which will be terminated at the Point D by the Arch ADB deſcrib'd from the Center C, thro' the Two ends A,B, and that Point D will be the Dividing Center of the Equinoctial AB, which may be divided into Hours by any of the foregoing Methods, without our repeating them again here. *Plato* 6. *Fig.* 25.

The Center of the Dial is thus found. Having at the Center of the Equator D made the Angle ODH of the Complement of the Height of the Pole, take upon the Meridian the Part OE, equal to the line DH, and the point E will be the Center of the Dial, from which you muſt draw the Hour-lines thro' the Points mark'd upon the Equinoctial.

To find the Foot of the Style and determine its Length, deſcribe about the Line OE, the Semi-circle EFO, and apply upon its Circumference the *Radius* of the Equator OF, equal to the Line OD. Laſtly, draw the *Axis* EF, and draw from the Point F to the Meridian CD, the Perpendicular FG, which will determine the Length of the Style, and give its Foot at the Point G.

DE-

DEMONSTRATION.

Plate 6. *Fig.* 25. If you draw the Line AD or BD, you will eaſily know that in the Iſoſceles Triangle ACD, the Angle ACD being of 30 Degrees, (*viz.* half of the Whole Angle ACB, which is of 60 Degrees, becauſe of the Equilateral Triangle ABC,) the Angle ADC is of 75 Degrees; ſuch as it muſt be, when the Point A is the Point of 5 a Clock, as we have ſuppos'd it; whence the Point D may be taken for the Dividing Center of the Equinoctial AB, in reſpect of which FG is the Length of the Style, becauſe the Line OD is equal to the Hypotenuſe OF, and the Angle EFG, or GOF is equal to the Complement of the Elevation of the Pole, becauſe it is equal to the Angle ODH, by reaſon of the Equality of the Two Rectangular Triangles DOH, OFE whoſe Sides OD OF are equal to one another, and alſo the Hypotenuſe DH, OE, *&c.*

PROBLEM IV.

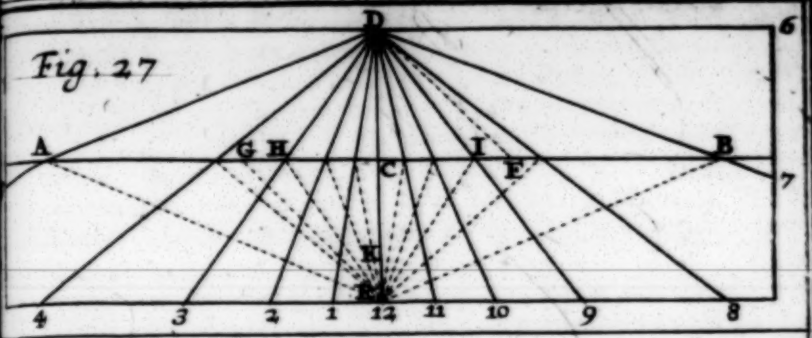
How to draw an Horizontal Dial, *beginning at the Points of* 5 *and* 7 *a Clock, mark'd upon the* Vertical Line.

IT is certain, that the Firſt Vertical is parallel to ſuch an Horizon of the Oblique Sphere, as has its Pole rais'd above it, the Complement of the Height of the Pole above the Horizon of the Place where you are: And that That Circle being perpendicular to a Horizontal-plain, and to the Meridian, muſt be repreſented upon ſuch a Plain by a Right-line, which we will call *Vertical Line*, which, *by Lem.* 1, 2. *Chap.* 1. will paſs thro' the Foot of the Style, and be perpendicular to the Meridian.

Plate 7. *Fig.* 27. Having then determin'd the Vertical AB of what Length you pleaſe, and having taken, as before, its End A for the Five a Clock Point, and the other End B for the Seven a Clock Point, divide that Vertical AB into Two equal Parts at C, which will be the Foot of the Style, and South-point, thro' which you muſt draw the Meridian DE perpendicular to the Vertical AB.

At A the Five a Clock Point make, with the Vertical AB, the Angle CAE of the Degrees of the Complement of the Horary Arch of Five a Clock, for the Complement of the Elevation of the Pole above the Horizon of the Place where you are, ſuch as it is found in the Table of the Horary Arches. As for *Paris*, whoſe Latitude is of about 49 Degrees, you will find in the Table of the Horary Arches, (over againſt

Fig. 27



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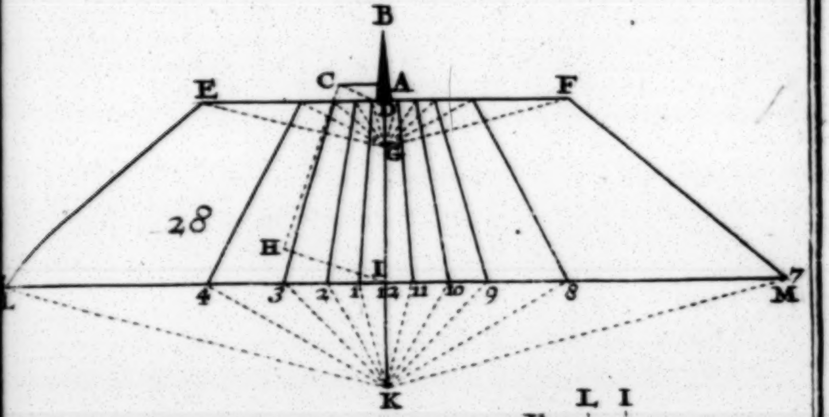

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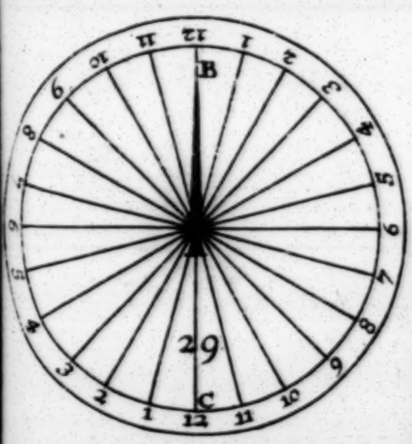

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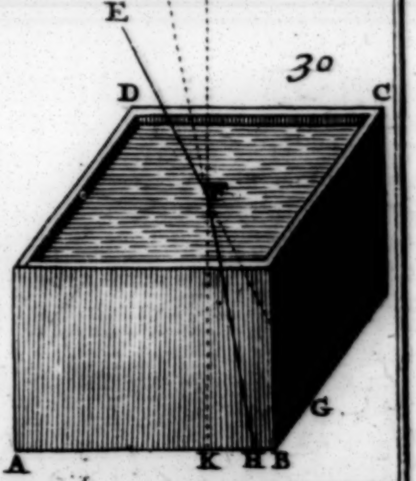

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41 Degrees, the Complement of 49) that the Horary Arch of Five a Clock is 67 Degrees and 47 Minutes, whoſe Comment is 22 Degrees and 13 Minutes for the Angle CAE. *Plate* 7. *Fig.* 27.

Having then at A, the Five a Clock Point, made the Angle CAE of 22 Degrees and 13 Minutes, you will have at E upon the Meridian DE the Dividing Center of the Vertical AB, which you muſt divide into Hours, by means of the Table of Horary Arches, (as the firſt Vertical is divided by the Horary Circles, which is, as we have ſaid, a Horizon, above which the Pole is rais'd by the Complement of the Latitude of the Place where you are) *viz.* making for *Paris* at that Dividing Center E, Angles with the Meridian agreeable to the Horary Arches, which are found in the Table over againſt Latitude 41, as of 9 Degrees, and 58 Minutes for 1 and Eleven a Clock, of 20 Degrees and 45 Minutes for 2 and Ten a Clock, *&c.*

Since the Point C is the Foot of the Style, the Line CE will repreſent its Length, which being ſet off upon the Vertical-line AB, from C to F, which will be the Dividing Center of the Meridian DE, you need only make at the Point F, the Angle CFD of the Complement of the Elevation of the Pole, to have at D the Center of the Dial, from which you muſt draw thro' the Horary-points of the Vertical AB, the Hour-lines, and the Dial will be deſcrib'd; whoſe Center D may alſo be found, by making at the Point E the Angle CEG of the Complement of the Height of the Pole, making the Line CD equal to the Line CG.

SCHOLIUM.

If inſtead of determining upon the Vertical AB, the Points A and B, of Five and Seven a Clock, you had determin'd the Points HI of Three and Nine a Clock, ſetting off the Diſtance CH, or CI, upon the Meridian at CK; you muſt conſider the Line AB as an Equinoctial-line, whoſe Dividing Center is K, by means of which it may be divided into Hours, according to all the Methods that have been taught; *in Probl.* 1. after which you will find D the Center of the Dial, as before.

PROBLEM V.

How to deſcribe an Horizontal Dial *without a* Center.

WHEN the Height of the Pole above the Horizon is but ſmall, the Center of the Dial will be very diſtant from the Foot of the Style, ſo that it will be difficult to mark it upon the Plain, unleſs you take a very ſhort Style, when *Fig.* 28.

Plate 7. Fig. 28. you wou'd do it without help of the Computed Tables. In ſuch a caſe the Dial may be drawn without a Center, thus:

Having drawn thro' A, the Foot of the Style, the Meridian AK, and having drawn to it thro' the Foot of the ſaid Style A, the Perpendicular AC, equal to the Length of the Style AB, draw at the Point C the Angle ACD of the Elevation of the Pole; and draw thro' the Point D the Equinoctial EF perpendicular to the Meridian AK. Set off CD the *Radius* of the Equator upon the Meridian from C to G, which will be the Dividing Center of the Equinoctial EF, which you muſt divide into Hours by ſome of the Methods of *Probl.* 1.

The Hour-lines might be drawn by making at the Hour-points mark'd upon the Equinoctial EF, Angles equal to the Complements of the Horary Arches, according to the Elevation of the Pole above the Horizon of the Place, ſuch as may be found in the Table of the Horary Arches, which we have given in *Lem.* 11. *Chap.* 1. But as you may want ſuch a Table, a ſecond Equinoctial may be made uſe of, thus:

To CD the *Radius* of the Equator, draw the Perpendicular CH, of what Length you pleaſe, which will repreſent part of the *Axis* of the Dial, and draw to it thro' the point H, the Perpendicular HI, which will be a ſecond *Radius* of the Equator, and will give upon the Meridian AK the Point I, thro' which you muſt draw to the ſaid Meridian the Perpendicular LM, which will be a ſecond Equinoctial, whoſe Dividing Center will be found upon the Meridian, by making IK equal to IH.

Laſtly, divide this ſecond Equinoctial LM into Hours, by the Precepts of *Probl.* 1. and join Two Points of the ſame Hour in each of the Two Equinoctials EF, LM, by Right-lines, which will be the Hour-lines, and the Dial will be deſcrib'd; to which, inſtead of the Style AB, you may fix perpendicularly a Piece like the Figure CDIH, on the Part HI, of the Meridian AK, and ſuch a Piece will ſerve for an Axis.

PROBLEM VI.

How to draw an Horizontal Dial *in the* Right Sphere.

AN Horizontal Dial deſcrib'd upon a Plain, parallel to the Horizon of the Right Sphere, is call'd a *Polar Dial*; becauſe its Plain paſſes thro' the Two Poles of the World, for which reaſon it has no Center, nor Six a Clock Line, becauſe in the Right Sphere the Sun Riſes and Sets at all times at Six of the Clock: And conſequently the Hour-lines are parallel to one another, and to the Meridian, and therefore perpendicular to the Equinoctial, which muſt go thro' the

Foot of the Style, *by Lem.* 1. becauſe the Equator is perpendicular to all the Horizons of the Right Sphere.

Having drawn thro' the Foot of the Style A, the Meridian BE, and having thro' the ſaid Foot A drawn CD perpendicular to it for your Equinoctial-line, ſet off the Length of the Style AB upon the Meridian BE, from A to E, which will be the Dividing Center of the Equinoctial CD, which muſt be divided into Hours by the Precepts of *Probl.* 1. and draw thro' the Horary-points mark'd upon the Equinoctial CD, the Hour-lines parallel to the Meridian BE, or perpendicular to the Equinoctial CD, and the Dial will be deſcrib'd, in which the Style AB may be fix'd to what Point you will of the Meridian BE, when it is only to ſhew the Hours: And if you wou'd have an *Axis*, raiſe perpendicularly upon the Meridian BE, a little Plain, like the Figure FG, whoſe Height muſt be equal to the Length of the Style AB. *Plate* 6. *Fig.* 26.

PROBLEM VII.

How to draw an Horizontal Dial *in the* Parallel Sphere.

AN Horizontal Dial, deſcrib'd upon a Plain parallel to the Horizon of the Parallel Sphere, is call'd an *Equinoctial Dial*, becauſe ſuch a Plain is parallel to the Equator, where A the Foot of the Style repreſents the Upper Pole, which is in the Zenith, for which Reaſon the Style AB has no determinate Length, in reſpect to the Hours, becauſe it repreſents the *Axis* of the World when it is perpendicular to the Plain: And as the Day is of 24 Hours, for half a Year, becauſe during that whole Time it does not Set in that Horizon, which repreſenting the Equator, is divided by the Horary Circles into 24 equal Parts; it follows, that if from A the Foot of the Style, a Circle be drawn at pleaſure, to repreſent the Equator, or One of its Parallels, and that Circle be divided into 24 equal Parts, beginning from the Meridian AC, and that from A, the Foot of the Style, as many Right-lines be drawn thro' the Points of the Diviſion; thoſe Lines will be the Hour-lines, and the Dial will be finiſh'd. *Plate* 7. *Fig.* 29.

PROBLEM VIII.

How to draw an Horizontal Dial *by Reflection.*

TO draw a Dial upon an Horizontal Plain, as, *for Example*, on a Cieling, which will ſhew the Hours by the Reflection of the Sun's Light, you muſt make uſe of an Horizontal Dial drawn according to the foregoing Method,

thod, upon a Plain truly Horizontal, *for Example*, upon a Window, or any other Plain which may be enlighten'd by the Rays of the Sun; but this Dial muſt have a contrary Situation to that which it ought to have when you wou'd ſhew the Hours, ſo that its Center, inſtead of being turn'd towards the South, muſt be turn'd towards the North.

This Horizontal Dial being thus fix'd with its Style, a Thread muſt be apply'd to what Point you will of each Hour-line, and extended till going thro' the end of the Style, it meets the Cieling in a Point which will be the Repreſentation of that Hour-point: And if the Thread be thus applied to any other Point of every Hour-line, you will upon the Cieling have another Point of that Hour-line from which the Thread is ſtretch'd, and ſo finding as many different Points as you pleaſe, you will finiſh the Dial.

In ſuch a Dial you will know the Hours by the Reflection of the Sun's Rays, by fixing at the end of the Style of the Horizontal Dial a little flat piece of Looking-Glaſs, whoſe Situation muſt be exactly Horizontal; but this will be done better if inſtead of a flat piece of Looking-Glaſs, you make uſe of Water, which of it ſelf will get into a Horizontal Situation, which is moſt eaſy and certain, becauſe if the Looking-Glaſs be ever ſo little Inclin'd, the Reflection will be twice as much chang'd; that is, if the Looking-Glaſs varies from its true Situation One Degree, *for Example*, the Reflection will vary by Two Degrees, which will cauſe a conſiderable Error.

Beſides when the Sky is not very Clear, or there are Clouds about the Sun, which cauſes the Looking-Glaſs to be but little enlighten'd by the Rays of the Sun, the Reflection cannot eaſily be diſtinguiſh'd upon the Cieling: Whereas the Water, which is almoſt always in Motion, (tho' ſcarce perceivable) by reaſon of the Agitation of the Air, gives a ſenſible Motion to the Light reflected on the Cieling, which makes that Point of Reflection more diſcernable, and therefore the Hour is more plainly ſhewn.

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If from the Center of the Looking-Glaſs, which repreſents the end of the Style of the Dial deſcrib'd upon the Cieling, a Thread be rais'd perpendicularly till it touches the Cieling in a Point, that Point will be the Place of the Style, and the Thread will repreſent its Length, by means of which may be drawn upon the Cieling a Horizontal Dial, by the foregoing *Problems*; but as thoſe kind of Surfaces are commonly rough and irregular, it is more proper to make uſe of the foregoing Method; which tho' merely Mechanical, has never fail'd us.

Gnomonicks or Dialling Plate 9 Pag: 65

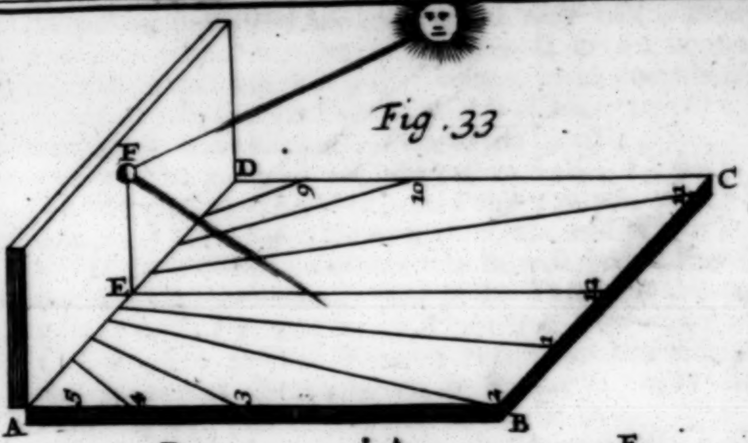
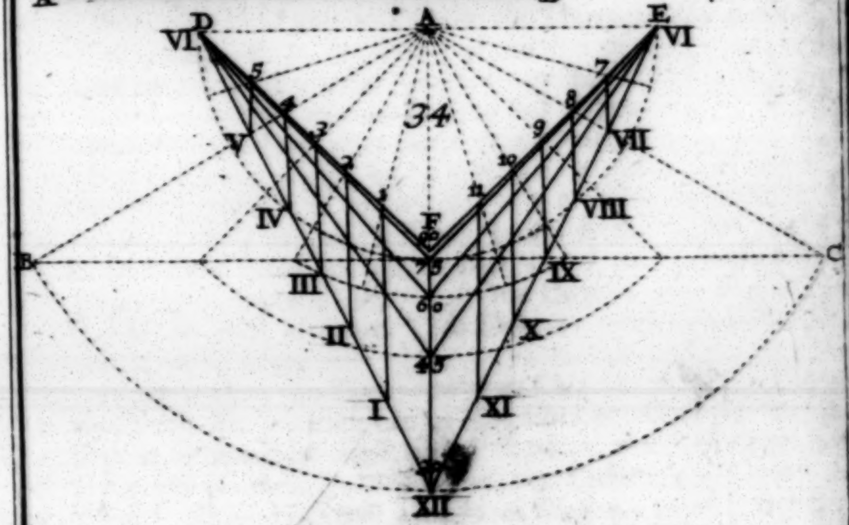


Fig. 33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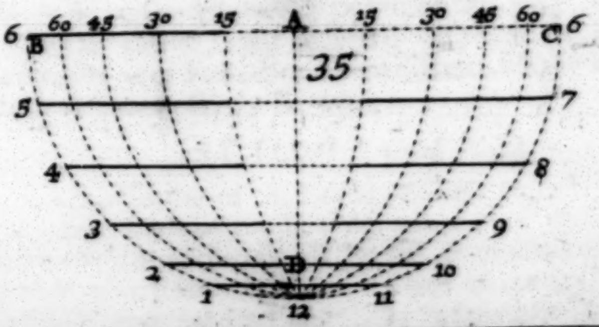
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Plate 9. *Fig.* 33.

It is plain, that after the ſame manner the Hours may be ſhewn by Reflection upon the upper Surface of an Horizontal Plain, as ABCD, by drawing upon it an Horizontal Dial; which muſt, as we have ſaid, be plac'd in a contrary Situation, and raiſing from E the Foot of the Style, the Perpendicular EF equal to the Style, to fix at the Point F a little flat piece of Looking-glaſs rais'd perpendicularly, from which the Rays of the Sun being reflected, will ſhew the Hours upon the Surface ABCD, which muſt be cover'd from the Sun, that the Reflected Light may appear, and ſhew the Hours upon the Horizontal Dial when 'tis ſhaded.

PROBLEM IX.

How to deſcribe an Horizontal Dial *by Refraction:*

WE have ſaid nothing concerning Reflection; becauſe it is what every Body underſtands: But ſince Refraction is not ſo well known, before we come to the Practice, it will not be improper to explain ſome Terms neceſſary to underſtand the Refraction which happens in all *Diaphanous Bodies*; that is, ſuch Bodies as can be penetrated by Light.

Plate 7. *Fig.* 30.

Experience teaches us that a Ray of Light, as EF, falling out of any Medium, as Air, upon a *Diaphanous* Body, ſuch as the Water contain'd in the Veſſel ABCD, that Ray EF, which is uſually call'd the *Ray of Incidence*, inſtead of going thro' the Water, according to its firſt Direction along the Right-line EFG, (as it wou'd do if there was no Water) turns off along the Right-line FH; and this is call'd *Refraction*, becauſe the Ray EF is broken, that is, turn'd out of its ſtreight Direction, running obliquely along the Line FG, which here comes towards the Perpendicular; that is, comes nearer to the Line IK, which is perpendicular to the Surface of the Water; becauſe the Ray EF paſſes out of a Rare, into a Denſer Medium.

Juſt the contrary wou'd happen, if the Ray EF ſhou'd go out of a Denſe, into a Rarer Medium; that is, in ſuch a caſe the Ray wou'd turn from the Perpendicular: As if HF was a Ray of Light, paſſing from Water into Air, inſtead of continuing to run in the Right-line HFL (as it wou'd do without the Reſiſtance of the Air) it will be broken and go obliquely along the Line EF, which runs from the Perpendicular IFK; whence this ſecond kind of Refraction is call'd *Refraction from the Perpendicular*, as the firſt is call'd *Refraction towards the Perpendicular*.

Whence it follows, That if the Eye be at E, it can ſee the Point H by the Oblique Line EFH, which will make the

Plate 7. *Fig.* 30.

Point H to appear higher than it really is. Thus it is no wonder that an Object, *for Example*, a piece of Money, which can't be ſeen in the Bottom of an empty Veſſel, by reaſon of the Height of its ſides, may be ſeen by pouring Water into the ſaid Veſſel, becauſe then there will be a Refraction ſufficient to raiſe the Object high enough to make it viſible, without changing the Place of the Eye.

The Line EF being a Ray of Incidence, the Line FH is call'd the *Ray of Refraction*, and the Angle GFH is call'd the *Angle of Refraction*. The Point F is call'd the *Point of Incidence*, and the Line IF which falls perpendicularly upon the Surface of the Water, is call'd the *Axis of Incidence*, and its continuation FG within the Water, is call'd the *Axis of Refraction*.

If you imagine a Plain to run along the Ray of Incidence EF, and the Ray of Refraction FH, this Plain is call'd the *Plain of Refraction*, which is perpendicular to the Surface of the Water, and conſequently paſſes along the *Axis* of Incidence, and the *Axis* of Refraction, and contains the Angle of Refraction GFH, and likewiſe the Angle KFH, which is call'd the *Refracted Angle*.

The Angle which the Ray of Incidence makes with the Surface of Water, is call'd the *Angle of Incidence*, and that which it makes with the *Axis* of Incidence IF, *viz*. EFI, is call'd the *Angle of Inclination*. The Refracted Angle becomes leſs as the Angle of Incidence becomes greater, ſo that it is reduc'd to nothing, when the Angle of Incidence is of 90 Degrees, in which caſe there is no Refraction.

Nevertheleſs the Refracted Angle does not increaſe in the ſame proportion as the Angle of Incidence decreaſes, but its Sine decreaſes in the ſame proportion as the Sine of the Angle of Incidence decreaſes; that is, the Sine of the Angles of Incidence in the ſame Medium are proportional to the Sines of their Refracted Angles in another Medium more Denſe, or more Rare; ſo that the Sine of an Angle of Incidence : is to the Sine of the correſpondent Refracted Angle :: as the Sine of any Angle of Incidence whatever: is to the Sine of the correſpondent Refracted Angle.

Wherefore having once known, by Experience, the Refracted Angle of any One Angle of Incidence, it is eaſy to find out by Trigonometry, the Refracted Angles of all the other Angles of Incidence: And according to this Method, the following Table has been calculated, whoſe firſt Column on the Left Hand contains the Degrees of the Angles of Inclination, to which anſwer in the ſecond Column towards the Right Hand, the Degrees and Minutes of the Angles Refracted in Water.

A Ta-

A Table of the Angles Refracted in Water.

D.	D.M.	D.	D.M.	D.	D.M.
1	0.46	31	25.38	61	42.52
2	1.33	32	24.21	62	43.23
3	2.20	33	25. 4	63	43.53
4	3. 7	34	25.47	64	44.21
5	3 54	35	26.30	65	44.50
6	4.40	36	27.13	66	45,17
7	5 27	37	27.55	67	45.44
8	6.13	38	28.35	68	46.20
9	7. 0	39	29.19	69	46.34
10	7.46	40	30. 0	70	46.58
11	8.32	41	30.42	71	47.21
12	9.18	42	31.22	72	47.43
13	10. 4	43	32. 2	73	48. 3
14	10.50	44	32.42	74	48.23
15	11.36	45	33.22	75	48 43
16	12.22	46	34. 2	76	49. 1
17	13. 9	47	34 42	77	49 17
18	13.55	48	35.19	78	49.33
19	14.40	49	35 57	79	49.47
20	15.25	50	36.35	80	50. 0
21	16.12	51	37.12	81	50.12
22	16.57	52	37.47	82	50.23
23	17.42	53	38.24	83	50 32
24	18.27	54	39. 0	84	50.41
25	19.12	55	39 35	85	50.48
26	19.56	56	40. 9	86	50.54
27	20.40	57	40.43	87	50.58
28	21.25	58	41.17	88	51. 1
29	22.10	59	41.49	89	58. 3
30	22.45	60	42.21	90	0. 0

This Table will ſerve to deſcribe a Dial in the Concavity of a Veſſel whoſe Poſition is Horizontal, which muſt not be very deep, that you may ſee in it what a Clock it is, when there is Water enough pour'd in to cover the end of the Style. But to deſcribe ſuch a Dial, you muſt make uſe of Two other Tables, ſuch as the Two following, of which the Second ſhews the Sun's Vertical, from the Meridian at every Hour of the Day, at the Beginning of each Sine, for the Latitude of 49 Degrees : And the Firſt Table ſhews the diſtance of the Sun from the Zenith, or the Complement of the Height of the Sun above the Horizon, at every Hour of the Day, every 10 Degrees of every Sign, for the ſaid Latitude of 49 Degrees.

A Table of the Diſtance of the Sun from the Zenith, every Hour of the Day, for the Latitude of 49 *Degrees.*

H.	XII.	XI.	X.	IX.	VIII.	VII.	VI.	V.
S.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D M.	D.M.
30.♋	25.30	28. 4	34.41	43.24	52.59	62.48	72.28	81.38
20.10	25.51	28.27	34.59	43.42	53:16	63. 4	72,48	81.56
10.20	26.58	29.29	35.56	44.32	54 1	63.52	73.38	82.48
♊ ♌	28.48	31.11	37.26	45.53	55.20	65. 9	74,53	84.10
20.10	31.12	33.30	39.31	47.46	57. 6	66.53	76.39	86. 3
10.20	34. 8	36.18	42. 3	50. 5	59.18	69. 2	78.48	88.20
♉ ♍	37.30	39.30	44.59	52.46	61.50	71.31	81.20	
20.10	41. 9	43. 2	48.16	55.47	64.41	74.17	84. 6	
10.20	45. 2	46.48	51.45	59. 0	67.42	77.12	87. 1	
♈ ♎	49. 0	50.40	55.23	62.22	70.51	80.13		
20.10	52.58	54.34	59. 2	65.45	74. 2	83.18		
10.20	56.51	58.20	62.36	69. 5	77. 9	86.16		
♓ ♏	61.30	61.56	66. 2	72.18	80.10	89. 6		
20.10	63.52	65.14	69. 9	75.15	82.55			
10.20	66.48	68. 8	71.55	77.48	85.18			
♒ ♐	69.12	70.30	74.12	79.57	87.19			
20.10	71. 2	72.16	75.54	81,33	88.48			
10.20	72. 9	73.22	76.57	82.33	89.42			
♑.30	72.30	73.45	77.18	82.52				
H.	XII.	I.	II.	III.	IV.	V.	VI.	VII.

A Table of the Verticals of the Sun, counting from the Meridian, at every Hour of the Day, for the Latitude of 49 *Degrees.*

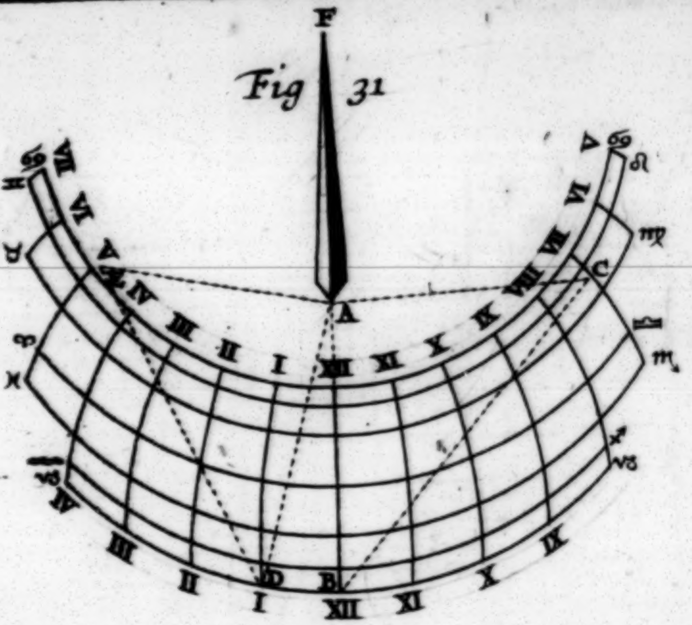
H.	XI.	X.	IX.	VIII	VII.	VI.	V.	IV.
S.	D.M.	D.M.	D M	D.M.	D.M	D. M	D. M	D. M.
♋	30.17	53.40	70.30	83.57	95.20	105.56	116.28	127.26
♌ ♊	27.58	50.33	67.34	81. 6	92.45	103.55	114.56	
♍ ♉	23 30	43 52	60.29	74.17	86.21	97.36		
♎ ♈	19 33	37.25	52.58	66.57	78.34			
♏ ♓	16.42	32.25	46.30	59,28	71.12			
♐ ♒	14.56	29.11	42.23	54.26				
♑	14.19	28. 2	40.48					
H.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.

Plate 8. *Fig.* 31. By means of theſe Two Tables, and of the foregoing, the propos'd Sun-Dial may be thus deſcrib'd. Draw thro' the Foot of the Style A, the Meridian AB, and draw to it thro' the ſame Point A, the Perpendicular AC equal the Style AF, which I ſuppoſe rais'd at Right Angles in the middle of the Bottom of the Veſſel. Then mark the Points of the beginnings of the Signs of the Zodiack upon that Meridian AB, thus:

To mark upon it, *for Example*, the beginning of ♑, look in the Second of the Three foregoing Tables, for the Diſtance of the Sun from the Zenith at Noon, when it is in ♑, and as that Diſtance is of 72 Degrees and 30 Minutes, for the Latitude of 49 Degrees, ſuppoſing that you wou'd mark your Dial for *Paris*, (whoſe Latitude being about ſo many Degrees) at the Point C there ſhou'd be made an Angle of 72 Degrees and 30 Minutes, but as that Angle is refracted in the Water, and chang'd into an Angle of 47 Degrees and 53 Minutes, that is, of about 48 Degrees, (as may be ſeen in the Table of Refractions) the Angle at C (*viz.* ABC) muſt made of 48 Degrees, to have upon the Meridian AB, the Point B of ♑. After the ſame manner muſt be mark'd all the Points of the Beginnings of the other Signs upon the ſaid Meridian AB.

But

Fig 31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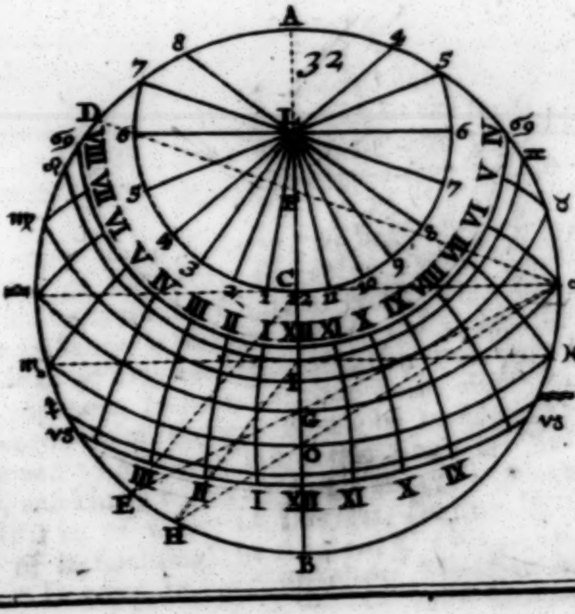
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But as the other Hour-lines, are not Right-lines, becauſe the Horary Circles which they repreſent are not perpendicular to the Plain of the Dial, which cauſes them to be bended and refracted in the Water, ſeveral Points of 'em muſt be found, and likewiſe ſeveral Points of the beginning of each Sign of the Zodiack, thus: *Plate* 8 *Fig.* 31.

To find (*for Example*) the Point of the Beginning of ♑, and of the One a Clock-line, look in the Table of the Sun's Verticals, for the Diſtance of the Sun from the Meridian at One a Clock, when it is at the Beginning of ♑; and as that Diſtance will be of 14 Degrees and 19 Minutes, at the Foot of the Style A, you muſt with the Meridian AB make the Angle BAD of 14 Degrees and 19 Minutes, with the line AD, which will repreſent the Sun's Vertical for the Time propos'd, upon which muſt be mark'd the Point of the firſt Hour, and that of ♑, thus:

Draw thro' the Foot of the Style A, the line AE perpendicular to the Vertical AD, and equal to the Style AF, and the Point E will be the Dividing Center of that Vertical AD: And becauſe the Table of the Diſtances of the Sun from the Zenith, ſhews, that the Sun, when in the beginning of ♑, is at One a Clock 73 Degrees and 45 Minutes diſtant from the Zenith, an Angle ought to be made at E of 73 Degrees and 45 Minutes, but as that Angle, by reaſon of the Refraction, is chang'd into an Angle of 48 Degrees and 18 Minutes, at the Point E you muſt make the Angle AED of 48 Degrees and 18 Minutes, to have at D the the Point of ♑ and of the firſt Hour.

After the ſame manner muſt be mark'd the Points of the other Signs and of the other Hours; then all the Points of the ſame Sign muſt be joyn'd by a Curve, which will repreſent the Parallel of that Sign; and likewiſe all the Points of the ſame Hour muſt be joyn'd by a Curve, which will repreſent the Hour-line and the Dial will be finiſh'd, which will ſhew not only the Hours, but the Sign in which the Sun is, by turning the Foot of the Style A towards the South, and pouring in Water enough to cover the Whole Style.

PROBLEM X.

How to deſcribe an Horizontal Aſtrolabe.

AN *Horizontal Aſtrolabe* is the Repreſentation of the Sphere upon the Plain of the Horizon, according to the Laws of the Stereographical Projection of the Sphere: It has only the Meridian and the other Vertical Circles repreſented by Right-lines, at leaſt in the Oblique Sphere, becauſe they are per-

Perpendicular to the Plain of Projection. Therefore theſe Circles will be very eaſy to deſcribe.

But without taking notice of all the Circles of the Sphere, we ſhall only teach here the way of repreſenting upon the Plain of the Horizon the Horary Circles, and the Parallels of the Signs for the Oblique Sphere, becauſe they ſuffice for what is requir'd, *viz.* to know the Hours by means of this Horizontal Aſtrolabe, whoſe Deſcription is as follows.

Plate 8. *Fig.* 32. Having drawn the Two Perpendiculars AB, ♎♈, (the Firſt of which AB muſt be the Meridian, and the other ♎♈ the Vertical, or Repreſentation of the Firſt Vertical,) from the Point C their common Section which repreſents the Zenith, deſcribe a Circle of what bigneſs you will, to repreſent the Horizon, which will be divided into Four equal Parts by the Two perpendicular Diameters AB, ♎♈.

Take upon that Horizon on One ſide the Arch AD, of the Elevation of the Pole; and on the other the Arch BE, of the Complement of the Height of the Pole, and draw from the Point ♈ thro' the Two Points D, E, the Two *Radii* ♈D, ♈E, which will upon the Meridian AB give One of the Poles of the World at F, and a Point of the Equator at G, thro' which and the Two Points ♎, ♈, which repreſent the Eaſt and Weſt Equinoctial Points, muſt be drawn the Arch ♎G♈, which will be the Repreſentation of the Equator.

For the Parallels of the Signs, draw the Line CE, and having taken the Arch EH of the Declination of the Parallel that you wou'd deſcribe, draw thro' the Point H, HI parallel to CE, and thro' the Point I, the Line ♏, ♓ perpendicular to the Meridian AB. Draw again from the ſaid Point H to the Point ♈ the Line ♈H, which upon the Meridian AB will give the Point O, thro' which and the Two Points ♏, ♓, you muſt draw the Circle ♏O♓, which will be the Repreſentation of the Parallel requir'd. And ſo on.

For the Repreſentation of the Horary Circles, take upon the Horizon on either Side, from the Points A and B, Arches equal to the Horary Arches which you find in the *Table of Horary Arches, page* 23. over againſt the Latitude of the Place where you are, and draw thro' the Pole F, and the Two Points of the ſame Hour Horary Circles, which will be the Repreſentation of ſuch Circles between the Tropicks as you have occaſion to mark, and the Dial will be finiſh'd; in which you will know the Hour, in Sun-ſhine, ſhewn in ſuch places as the Shadow of a Style perpendicularly erected at the Center C, will cut the Sun's Parallel, when the Point A is turn'd directly South.

SCHO-

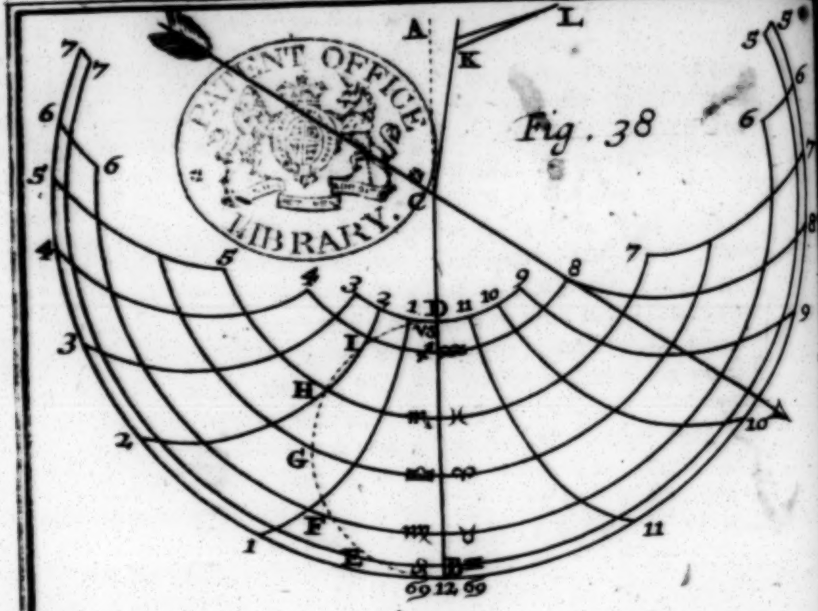

Fig. 38

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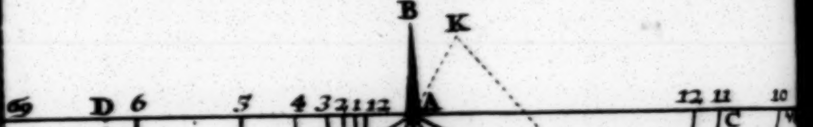

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SCHOLIUM.

If an Horizontal Dial be deſcrib'd above the Center C upon the ſame Meridian AB, whoſe Center is (*for Example*) L; the Style rais'd at Right-angles at the Center C, and the Axis at the Center L rais'd up to an Angle equal to the Height of the Pole, will, by their Shadow when the Sun-ſhines, ſhew the Hour, when the Center L is turn'd directly South, which will be in that Poſition when thoſe Two Shadows ſhew the ſame Hour, which will be the Hour requir'd.

If the Horizontal Aſtrolabe be deſcrib'd in a Square, whoſe Two Sides are parallel to the Meridian AB, it will ſhew the Hours without any Style, if inſtead of the Style you apply to the Center C a Magnetick-Needle, pois'd upon a Braſs or Steel-Point upon which it may turn freely: For if you turn the Plain of the Dial till One of the Two Sides, which are parallel to the Meridian, faces the Sun exactly (which it will do when it ceaſes to be enlighten'd) ſo that the Point A may be towards the Sun, the Needle will be inſtead of the Sun's Vertical, and ſhew the Hour upon the Sun's Parallel. This may alſo be done upon another Dial whoſe Conſtruction follows in

PROBLEM XI.

How to deſcribe an Azimuth Dial.

WE call this an *Azimuth Dial*, becauſe it is drawn upon an Horizontal Plain, by means of the Azimuths or Verticals of the Sun, which you find calculated for every Hour of the Day, and for the beginning of Each Sign in the Table of the Sun's Verticals, which is as we have ſaid for the Latitude of 49 Degrees, which is pretty near the Latitude of *Paris*.

Plate 12. *Fig*. 38.

Having thro' the Foot of the Style C, drawn the Meridian AB, of what length you will, and having from the ſaid Foot of the Style C deſcrib'd thro' the End B an Arch, which you muſt take for the Tropick of ♋, mark upon that Tropick the Sun's Verticals for each Hour of the Day, taking from B the Point of the Meridian, on either ſide, Arches of as many Degrees as you will find over againſt ♋ under each Hour in the Table of the Sun's Verticals, which you will have in Page 70.

Make CD about equal to the Third Part of BC, and deſcribe from C for your Center, thro' the Point D another Arch

Plate 12. *Fig.* 38. Arch, which you muſt take for the Tropick of ♑, upon which you muſt mark the Hour-Points, by means of the Sun's Verticals, as was done before upon the Tropick of ♋, which goes thro' the Point B.

Then deſcribe upon BD, the Semi-circle BGD, which you muſt divide into Six equal Parts at the Points E, F, G, H, I, thro' which you muſt from the ſaid Center C deſcribe Arches which will repreſent the beginnings of the other Signs of the Zodiack, upon which you muſt after the ſame manner mark the Hour-Points by means of the Table of the Sun's Verticals.

Laſtly, joyn all the Points of the ſame Hour by Curves drawn ſmooth without any Angles, which Curves will be the Hour-lines, and the Dial will be finiſh'd; in which you will know the Hours as in the foregoing, *viz.* by means of a Style rais'd at Right Angles at the Point C, which muſt be turn'd to the South, or by means of the Style KL rais'd perpendicularly on the Meridian, or on the Line CK, which ſhews the Declination of the Needle; for if you turn the Plain of the Dial till the Shadow of the Style KL covers part of the Line CK, the Magnetick Needle will ſhew the Hour upon the Sign in which the Sun is at that time.

PROBLEM XII.

How to deſcribe an Horizontal Dial *by means of the Sun's Altitudes.*

Fig. 39. Having drawn thro' the Foot of the Style A, whoſe Length AB muſt be but ſmall, the Right-line ♋ ♑, Part of which as A ♋ muſt be taken for the Tropick of ♋, and the other A♑ for the Tropick of ♑, deſcribe from the ſaid Foot of the Style A, the Semi-circle CGD, and divide it into Six equal Parts at the Points E,F,G,H,I, thro' which you muſt draw from the Center A as many Right-lines, which will repreſent the Parallels of the other Signs, upon which you muſt mark the Hour-points, by means of the Table of the Diſtance of the Sun from the Zenith, or of the Complements of the Sun's Altitudes, thus.

To mark, *for Example*, the South Point upon the Line AL, which repreſents the beginning of ♒ and of ♐, which muſt be conſider'd as the Sun's Vertical, draw to that Line AL, thro' the Foot of the Style A, the Perpendicular AK equal to the Style AB, and at the Point K make the Angle AKL of 69 Degrees and 12 Minutes, for the Diſtance of the Sun from the Zenith at Noon, ſuch as you find it in the Table of the Diſtances

Diſtances of the Sun from the Zenith over-againſt the Signs ♒, ♐.

After the ſame mannner will the Points of the ſame Hour be mark'd upon all the other Signs, which you muſt join by Curves which will repreſent the Hour-lines, and the Dial will be finiſh'd ; where you will know what of the Clock it is when the Sun Shines, by turning the Foot of the Style A towards the Sun, in ſuch manner that the Shadow of the Style ſhall fall upon the Sign in which the Sun is, and the End of the Shadow will ſhew the Hour requir'd.

PROBLEM XIII.

How to turn a Sun-Dial, *drawn for any particular Latitude, into an* Univerſal Dial.

IF you wou'd have an Horizontal Dial deſcrib'd for a particular Latitude, as *for Example*, for the Latitude of *Paris*, which is of about 49 Degrees, to ſerve for any other Latitude of more or fewer Degrees, the Plain of the Horizontal Dial muſt be ſo contriv'd, as to be rais'd above the Horizon of the Place where you are(towards the North if the Latitude of the Place where you are, is leſs than that of *Paris*, or towards the South if the Latitude of the Place where you are be greater than that of *Paris*.) by the Difference of the Latitude of the Two Places; For thus the Plain of the Dial will be parallel to the Horizon of *Paris*, and conſeqently may ſhew the Hours of the Place where it ſtands.

After this manner the Equinoctial, and Polar Dials are made Univerſal: But there is another eaſy way of making an Horizontal Dial, drawn for any particular Latitude, to become Univerſal without inclining its plain, or having any Hour-lines drawn upon that Plain, if the Hour-points be but mark'd upon the Equinoctial Line, thus:

Plate 6. Fig. 25.

Having, *by Probl.* 1. divided the Equinoctial AB into Hours, and having ended the reſt as has been taught in *Prob.* 3. you muſt at the Point O (where the Equinoctial and Meridian interſect(erect a ſmall perpendicular Plain Similar to the Rectangular Triangle OFE, and moveable about the Point O, in ſuch manner that the ſide OF, ſhall make with the Meridian OE which muſt be ſlit in that Place, an Angle equal to the Complement of the Height of the Pole above the Horizon of the Place where you are, which may be eaſily done; for thus the Axis EF will always be parallel to the Axis of the World, and by its Shadow ſhew the Hours upon the Equinoctial AB, when it is turn'd directly towards the South.

PROBLEM XIV.

How to deſcribe an Univerſal Horizontal-Dial.

Plate 13. *Fig.* 40. HAving thro' the Center of the Dial C, drawn the Two Perpendiculars AB, CD, (the Firſt of which AB being taken for the Line of Six a Clock, the Second CD will be the Meridian) ſet off upon that Meridian taken at Pleaſure, from D, towards XII, as many equal parts as you pleaſe, *for Example*, Six; deſcribe from the Center C thro' the Points of Diviſion, Arches which will repreſent the Circles of Latitude, which you may draw at every 5 Degrees, becauſe you may by the Eye judge of any Number between.

If you ſuppoſe the leaſt Circle which goes thro' the Point D, to be the Circle of Latitude for 60 Degrees, the Laſt and Greateſt which paſſes thro' the Point XII will repreſent the Circle of Latitude of 30 Degrees, upon which, as alſo upon all the others, you muſt on either ſide take from the Meridian CD, the Horary Arches according to the Latitude of each, as they may be found in the Table of the Horary Arches, page 23.

Thus you will have a Point of the ſame Hour upon each Circle of Latitude, which you muſt joyn neatly by Curves, which will be the Hour-lines, and the Dial will be finiſh'd; in which the Hours will be ſhewn where the Shadow of the Axis, (which at the Center C muſt be rais'd ſo as to make an Angle equal to the Height of the Pole above the Horizon of the Place where you are) cuts the Circle of Latitude of the ſame Place, when the Center C is directly oppoſite to the Rais'd Pole.

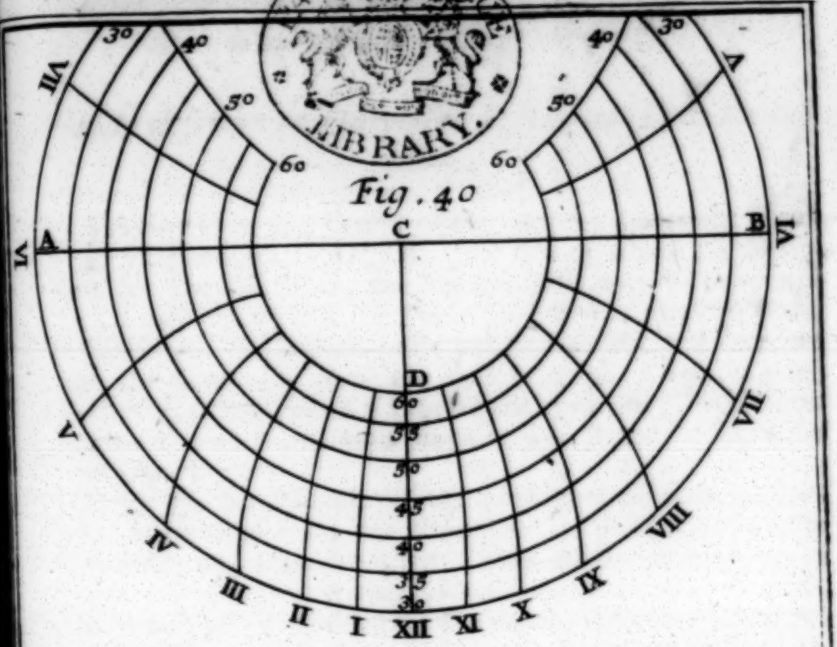

Fig. 40

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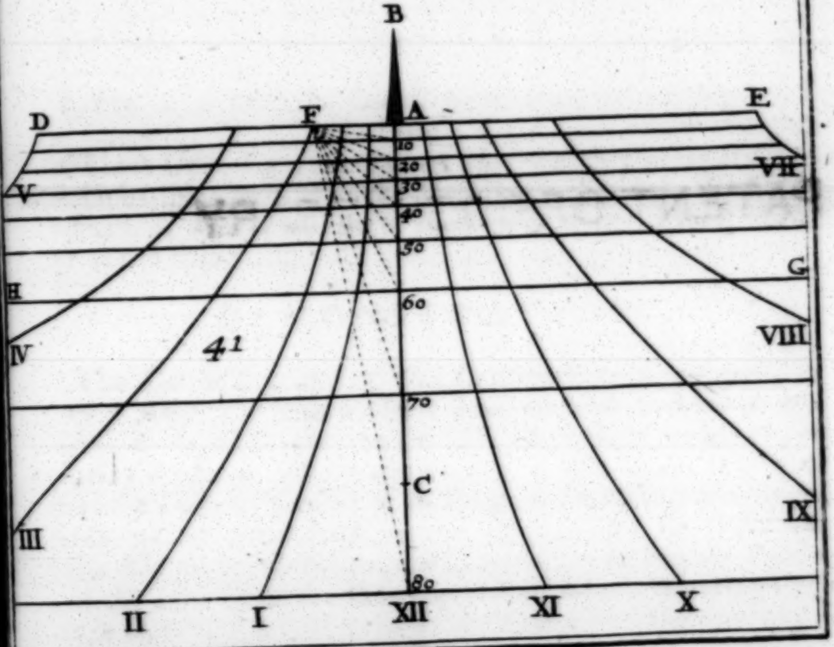

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PROBLEM XV.

How to deſcribe an Univerſal Rectilineal Horizontal Dial.

HAving thro' the Center of the Dial A, drawn the Two *Plate* 9. Perpendiculars AF, DE, (the firſt of which AF being *Fig.* 34. taken for the Meridian, the ſecond DE will be the Six a Clock Line) draw thro' the Point 90, taken at pleaſure upon the Meridian, the Indefinite Perpendicular BC, and deſcribe from the Center A thro' the ſaid Point 90, a Semi-Circle, which will upon the Six a Clock Line give the Two Points D,E, thro' which you muſt draw to the Point 90 the Two Lines D90, E90.

Divide the Semi-Circle D90E into Parts of 10 or 15 Degrees, beginning from the Point 90, or from One of the Two E,D, and draw from the Center A thro' the Points of Diviſion, Right-lines which will cut the Line BC in Points, thro' which you muſt deſcribe from the ſaid Center A, Arches which will cut the Meridian at the Points 75, 60, 45, 30, thro' which you muſt draw to the Two Points D.E, as many Right-lines, which will repreſent the Circles of Latitude for every 15th Degree.

Laſtly, draw from the Center A, thro' each 15th Degree of the foregoing Semi-Circle D90E, Right-lines which will cut the Lines D90, E90, in Points thro' which you muſt draw the Hour-lines parallel to the Meridian AF; and the Dial will be finiſh'd, which will ſhew the Hours after the ſame manner as the foregoing.

DEMONSTRATION.

The Demonſtration of this Conſtruction is evident, *by* *Plate* 2. *Lem.* 6. *Chap.* 1. where we have demonſtrated that the Angle *Fig.* 7. HAC is equal to the Angle CDE, which is equal to the Horary Arch, as will be known if you look upon the Point A as the Pole, or Center of an Equinoctial Dial; the Angle AGF, or its equal ABC, as the Elevation of the Pole, the Line AH as the Meridian, the Line AE as an Horary Circle, and the LineCE as the Horizontal-line, whoſe Dividing Center is D, *&c.*

SCHOLIUM.

This kind of Sun-Dial is call'd *Univerſal*, becauſe it ſerves univerſally for all the Places of the Earth, whoſe Latitudes are

are repreſented in it by Right-lines. The Hour-lines only are parallel to one another; but it may be ſo contriv'd that the Lines of Latitude ſhall alſo be parallel, if you look upon 'em as ſo many different Equinoctials, thus:

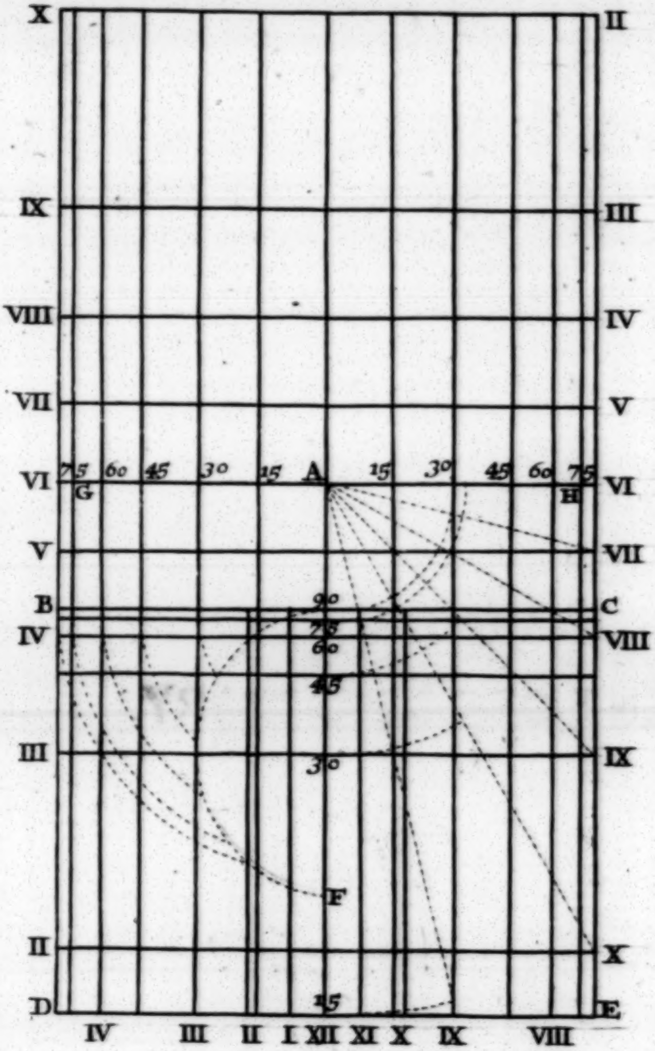
Having, as before, drawn thro' the Center of the Dial A, the Two Perpendiculars AF, GH, (the Firſt of which (*viz.*) AH being taken for the Meridian, the Second GH will repreſent the Six a Clock-line) draw thro' the Point 90 taken at pleaſure upon the Meridian AF, to the Six a Clock-line, the Parallel BC of ſuch a Length that each of the Lines 90B, 90C, may be double the Line A90, and draw thro' the Two Points B, C, the Right-lines BD, CE, parallel to the Meridian AF.

Take upon the Meridian AF, the Line 90F equal to the Line 90C, or 90B, that is, twice the Line A90, and from its middle Point 30 deſcribe thro' the Points F, 90, a Semicircle, which you muſt divide into parts of 10 or 15 Degrees, and ſet off the Chords of the Arches of 15 Degrees, of 30, of 45, *&c.* upon the Line BC on either ſide from the Point 90, at Points, thro' which you muſt draw Parallels to the Meridian AF, which will repreſent Circles of Latitude in reſpect to the Hour-lines, which muſt be drawn parallel to the Six a Clock-line, thro' the Diviſions of the Line BD, or CE, whoſe Dividing Center is A.

Deſcribe from the Center A, thro' the Point 90, the Quadrant 90,30, which you muſt likewiſe divide into parts of 10 or of 15 Degrees to draw from the ſaid Center A, thro' the Points of Diviſion, Right-lines as far as the Circle of Latitude of 30 Degrees, which is the ſame as the Nine a Clock-line, and ſet off the Length of thoſe Lines or Secants downwards upon the Meridian, from the Center A to the Points 75, 60, 45, 30, 15, thro' which you muſt draw Lines parallel to GH the Six a Clock-line, which will repreſent other Circles of Latitude in reſpect to other Hour-lines, which muſt be drawn parallel to the Meridian AF, thro' the Diviſions of the Line BC, whoſe Dividing Center is A, and the Dial will be finiſh'd; in which you may know what a Clock it is when the Sun Shines, after the ſame manner, as in the Two foregoing.

Gnomonicks or Dialling Plate 10 Pag. 78

Fig. 36



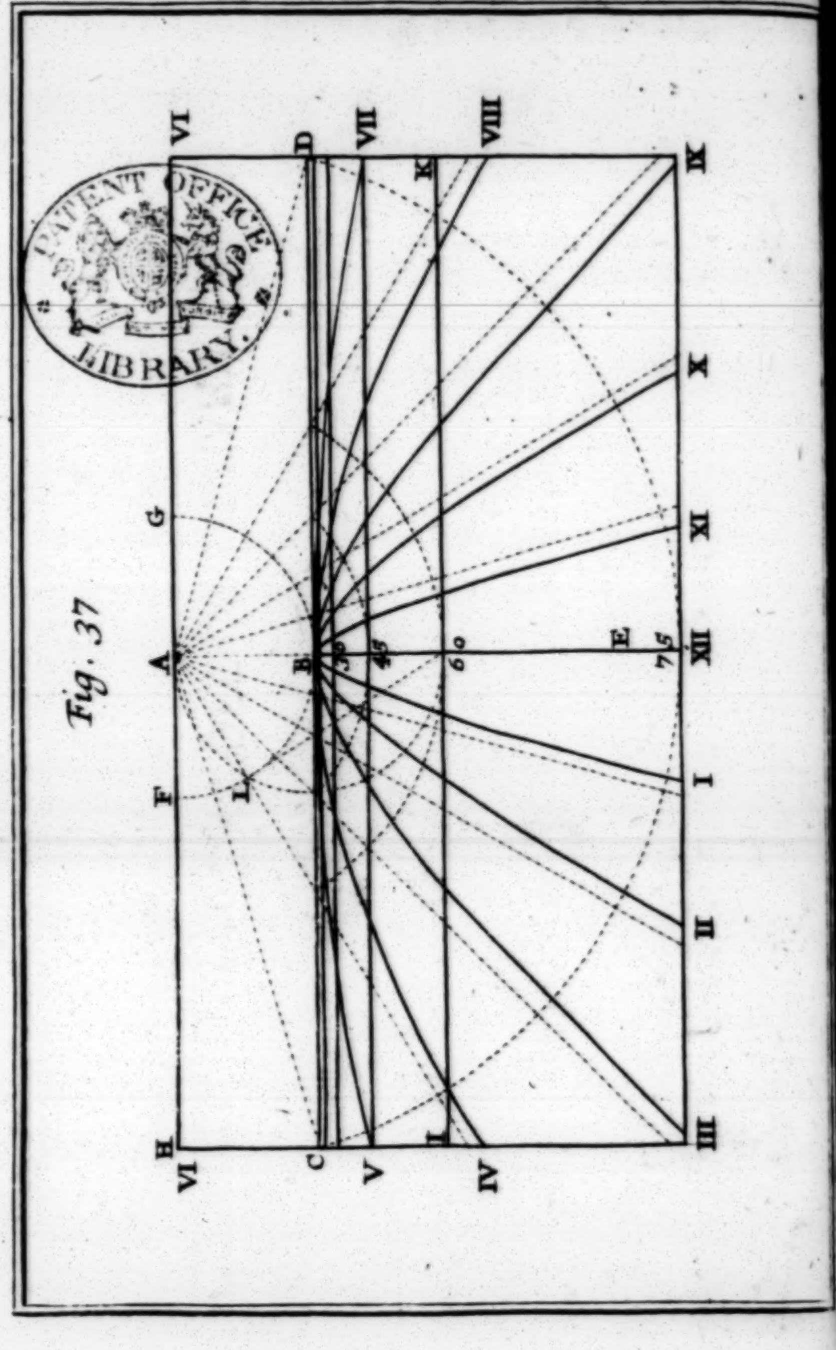


Fig. 37

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PROBLEM XVI.

How to draw an Univerſal Elliptical Horizontal Dial.

THIS Dial is call'd *Elliptical*, becauſe it is made according to the Principles of the Orthographical Projection of the Sphere, where the Circles which are not perpendicular to the Plain of the Projection, are repreſented by *Ellipſes*. Its Conſtruction will be eaſy for any one who has rightly underſtood what we have Taught in *Problem* 2. *Fig.* 23. Wherefore we ſhall give it here in a few Words.

Plate 9. *Fig.* 35.

Having drawn thro' A the Center of the Dial, the Two Perpendiculars AD, BC, (the firſt of which AD being, as before, taken for the Meridian, the Second BC will repreſent the Six a Clock-line) deſcribe from the ſaid Center A, on the Line BC the Semi-Circle B12C, and divide it into 12 equal Parts, beginning from the Six a Clock-line, or from the Meridian, to draw thro' the oppoſite Points of Diviſion which are equally diſtant from the Six a Clock-line BC, ſtreight Lines parallel to one another, and to the Line BC, which will repreſent the Hour-lines.

Mark upon thoſe Parallels (which are conſider'd as Circles of Latitude for every 10, or every 15 Degrees) the Hour-Points by the means of ſeveral Axes drawn from A the Center of the Dial, as has been Taught in *Probl.* 2. *Fig.* 23. Join all thoſe Horary Points which belong to the ſame Latitude by a Curve, which will be the Circumference of an Ellipſis, and repreſent the Circle of that Latitude, and the Parallel Lines will (as we have already ſaid) repreſent the Hour-lines, upon which you may, when the Sun ſhines, know what a Clock it is, by the ſame means as in the foregoing Dial.

PROBLEM XVII.

How to draw an Univerſal Hyperbolical Horizontal Dial.

WE call an *Hyperbolical Dial* an Univerſal Dial, in which all the Hour-lines are Hyperbola's, and the Lines of Latitude Right-lines. The following Method to deſcribe it is very eaſy.

Plate 11. *Fig.* 37.

Having, as before, drawn thro' A the Center of the Dial the Two Perpendiculars AE, HO, (the Firſt of which AE will

Plate 11. *Fig.* 37. will be the Meridian) deſcribe at pleaſure from that Center A the Semi-Circle FBG, which you muſt divide into 12 equal Parts, beginning from B the South Point, or from one of the Points F, G, and draw from the ſame Center A, thro' the Points of Diviſion Indefinite Lines, between which as Aſymptotes, you will deſcribe thro' the Point B, taken at pleaſure upon the Meridian AE, as many Hyperbola's to repreſent the Hour-lines.

If to the Meridian AE you draw thro' the Point B the Perpendicular CD, it will be divided by the foregoing Aſymptotes at every 15 Degrees into Points, thro' which, from the Center A, Arches muſt be drawn, which will upon the Meridian AE give other Points, thro' which muſt be drawn as many Lines Parallel to one another, and to the Line CD, which will repreſent the Circles of Latitude for every 15th Degree; upon which you will know the Hour of the Day after the ſame manner as in the foregoing Dials.

SCHOLIUM.

As theſe Lines of Latitude repreſent Equinoctial Lines according to their Latitudes, one may upon each of thoſe Lines find the Hour-points by means of their dividing Centers, which muſt be thus found. For Example, to find the Dividing Center of the Equinoctial IK, which repreſents the Circle of Latitude of 60 Degrees, deſcribe on the Line A 60, the Semicircle AL 60, which is cut at the Point L by the *Radius* AL of 60 Degrees, and ſet off the Diſtance or *Radius* of the Equator 60 L upon the Meridian from the Point 60 to the Point 75, which will be the Dividing Center requir'd, as it is evident by what has been demonſtrated in *Probl.* 3.

You will after the ſame manner find the Dividing Centers of the other Equinoctials, and by their means the Hour-points upon thoſe Equinoctials, by the ſhort Methods which have been Taught in *Probl.* 1. and joining all the points of the ſame Hour by Curves, you will have Hour-lines, which will be all Hyperbola's, (having for their common Center, the Center of the Dial A) of which the Line of 3 and 9 a Clock is an *Equilateral Hyperbola*, that is an Hyperbola, one Diameter of which is equal to its Parameter. Thoſe that underſtand Algebra, Conick Sections, and Geometrical *Loci*, will eaſily find the Demonſtration of this.

Plate 13. *Fig.* 41. We have another Method for deſcribing an Univerſal Hyperbolical Horizontal Dial, which we ſhall here explain in few Words, becauſe its Demonſtration goes along with it.

Having thro' the Foot of the Style A, drawn the Two Perpendiculars AC, DE, (the Firſt of which AC being taken for

for the Meridian, the Second DB will be the Horizon of the Right Sphere,) take upon that Horizon the Line AF, equal to AB, the Length of the Style, and from the Point F as a Dividing Center, divide the Meridian AC into Parts of 5, or 10 Degrees, to draw thro' the Points of Diviſion as many Perpendiculars to the ſaid Meridian AC; which will repreſent the Circles of Latitude for every 10th Degree, and which are as ſo many Equinoctial Lines, which may be thus divided into Hours. *Plate* 13. *Fig.* 41.

For Example, To divide into Hours the Equinoctial GH, which repreſents the Circle of Latitude for 60 Degrees, ſet off the Hypotenuſe, or Radius of the Equator F60, from 60 to the Point C upon the Meridian, which will be the Dividing Center of the Equinoctial GH, which may conſequently be divided into Hours, or into Parts of 15 Degrees, by the Precepts of *Probl.* 1.

The Hours muſt be divided after the ſame manner in the other Equinoctials, and if all the Points of the ſame Hour be join'd by Curves, they will repreſent the Hour-lines, which will be Hyperbola's, and the Dial will be finiſh'd; in which you will know the Hours at the Equinoctial Times, by means of the End of the Shadow of the Style AB, and at another Time by means of an Axis, which muſt be inclin'd at B, the end of the Style AB, in ſuch manner, that it may with the Style AB make an Angle equal to the Complement of the height of the Pole above the Horizon of the Place where you are. In this Dial, as in the foregoing, there are only the 3 and 9 a Clock-lines which are Equilateral Hyperbola's.

PROBLEM XVIII.

How to draw an Univerſal Parabolical Horizontal Dial.

WE call a *Parabolical Dial* an Univerſal Dial, where all the Hour-lines are Parabola's, and the Lines of Latitude Right-lines. It is deſcrib'd almoſt after the ſame manner as the foregoing, as you will ſee.

Having drawn thro' A, the Center of the Dial, the Two Perpendiculars A75, CD, (the Firſt of which A75, being taken for the Meridian, the Second CD will be the Six a Clock-line) deſcribe from the Center A, with what diſtance you pleaſe, the Semi-Circle C75D, and divide it into Parts of 10 or 15 Degrees, beginning from the Meridian, or from the Six a Clock-line, to join the Two oppoſite Points, which are equally diſtant from the Six a Clock-line, or from the Meridian *Plate* 14. *Fig.* 42.

Plate 14. *Fig.* 42. dian, by Indefinite Right-lines, which will be parallel to the Six a Clock-line, and conſequently perpendicular to the Meridian, and which will repreſent the Circles of Latitude, upon which you muſt mark the Hour-points, thus.

To divide into Hours (*for Examp.*) the Line GH of 30 Degrees of Latitude, draw from the Point 30 upon the Line AE, (which makes with the Meridian AF, an Angle of 30 Degrees) the Perpendicular 30 E, whoſe length being ſet off upon the Meridian, from 30 to the Point F; that point F will be the dividing Center of the Line GH, conſider'd as an Equinoctial, which you will divide into Hours by the Precepts of *Probl.* 1.

After the ſame manner muſt the Hour-points be mark'd upon the other Lines of Latitude, and then be join'd as before by Curves, which will repreſent the Hour-lines and be Parabola's, of which the 3 and 9 a Clock-lines have the *Radius* AC, or AD for their Parameter. Thus will the Dial be finiſh'd, which will ſhew the Hours by means of an Axis, which at the Center A muſt make an Angle Equal to the height of the Pole, above the Horizon of the Place where you are.

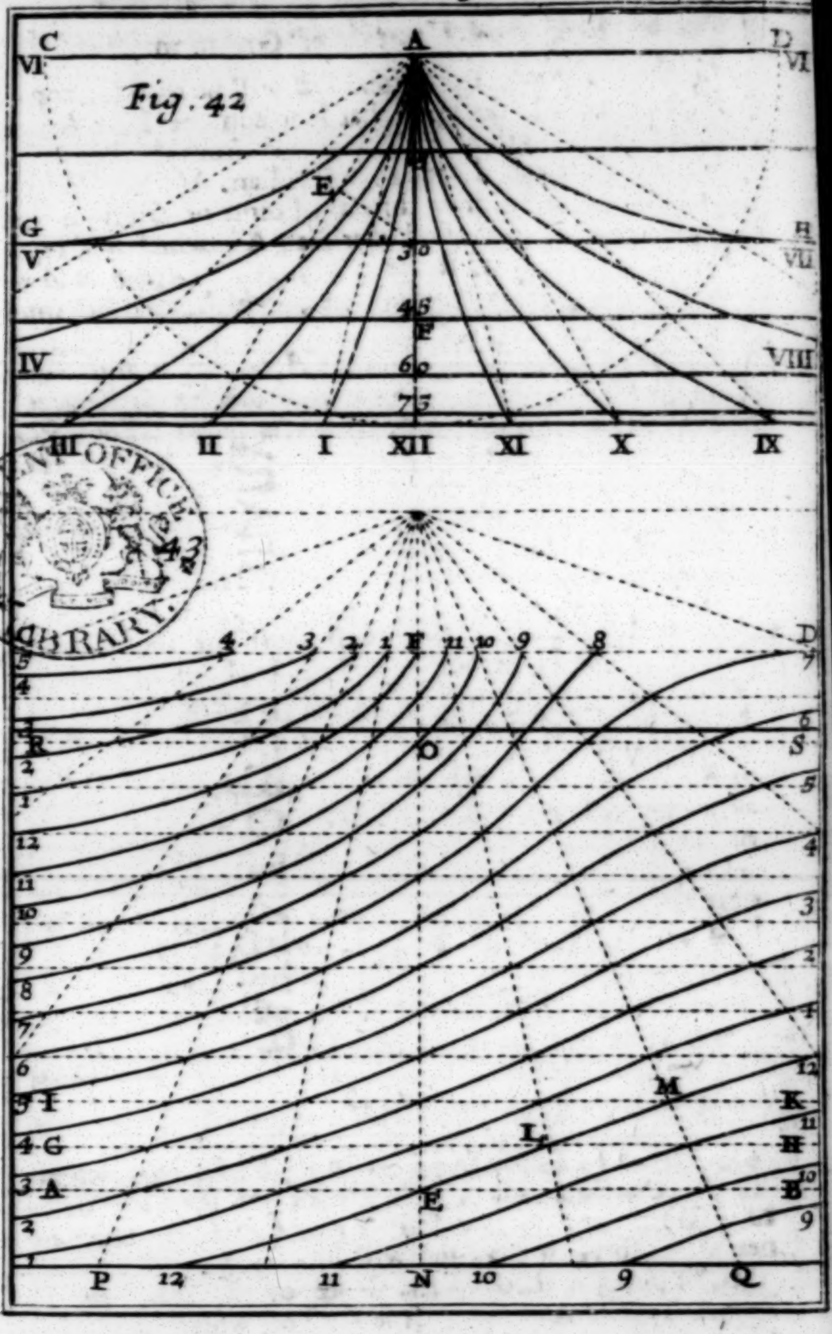
PROBLEM XIX.

How to draw an Horizontal Dial *for a particular Latitude, which will ſhew the Hours for every Place of the Earth.*

Fig. 43. HAving deſcrib'd an *Horizontal Dial* for the Latitude of the Place where you are; *for Example*, for *Paris*, draw at pleaſure the Two Lines AB, CD, parallel to one another, and perpendicular to the Meridian EF, and divide their Interval EF into 12 equal parts, to draw thro' the Points of Diviſion other Lines perpendicular to the Meridian, or parallel to the Two Firſt AB, CD, which will repreſent Meridians, diſtant from one another 15 Degrees or One Hour.

If you take the Firſt AB for the Meridian of *Paris*, and the next GH for the Meridian Eaſtward from that of *Paris*, and 15 Degrees, or One Hour diſtant, and the next IK for a Meridian 30 Degrees, or 2 Hours Eaſtward from *Paris*, and ſo on; the Point E being the South-point in AB, the Meridian of *Paris*, the Point L in GH, the following Meridian will alſo be the South Point; tho' the Line of 11 a Clock in the Morning paſſes thro' it; becauſe ſuch as are One Hour Eaſt from *Paris* have their Noon when it is but 11 at *Paris*. And likewiſe the Point M in the Meridian IK will be the South Point,

Gnomonicks or Dialling Plate 14 Pag.



Point, tho' the Line of 10 in the Morning paſſes thro' it ; because thoſe which are Two Hours Eaſt from *Paris*, have their Noon when it is but 10 at *Paris*. Wherefore all the Points E, L, M, and the other which belong to the South, are to be join'd by a Curve, which will repreſent a Meridian fit to ſhew when it is 12 a Clock in all places of the Earth, of known Longitude : Much after the ſame manner muſt the other Hour-lines be drawn, which you will eaſily underſtand by a Sight of the Figure. *Plate* 14. *Fig.* 43.

The Meridians, or Circles of Longitude are repreſented here but for every 15th Degree ; but as they are not counted here from the Firſt Meridian, and as in Maps they are drawn at every 10th Degree, if you wou'd have them ſo, (which will be more convenient for your ſetting down the Names of Places in your Dial, according to their Longitudes and Latitudes, as they are in the general Maps) Work thus.

Take upon the Meridian EF, the part EN of 22 Degrees and 30 Minutes, ſuch as is the Longitude of *Paris*, which you will do by dividing a 12th Part of the Interval EF, into 15 equal parts, which will repreſent Degrees, and draw thro' the Point N to the Meridian EF, the Perpendicular PQ, which will repreſent the firſt Meridian. Take alſo the part FO, equal to the part EN, and draw thro' the Point O to the ſaid Meridian EF, the Perpendicular RS, which will repreſent a Meridian, 180 Degrees, or 12 Hours diſtant from the firſt PQ; becauſe the Space NO is equal to the Space EF, by reaſon of the Two equal Arches EN, FO. Wherefore to have Longitude for every 10th Degree, from the Firſt Meridian, wipe out all the Lines perpendicular to the Meridian, which repreſent the Meridians for every 15th Degree, except the Two laſt PQ, RS, and divide their Interval NO into 18 equal Parts, at Points thro' which you muſt draw other Lines perpendicular to the Meridian EF, which will repreſent the Circles of Longitude for every 10th Degree, counting Eaſtward from the Firſt Meridian PQ, and you may, if you will, produce 'em either way from that Firſt Meridian, according to the Extent of the Country which you wou'd repreſent upon the Plain of the Dial

To deſcribe the Circles of Latitude, let the Meridian NO be your Equator, and divide each of the Two Parts OR, OS, which determine the Breadth of the Dial, into 9 equal Parts, each of which will repreſent 10 Degrees, to draw thro' the Points of Diviſion Lines parallel to the Equator ON, which will repreſent Circles of Latitude for every 10th Degree ; then ſet down the Number of Degrees at every 10th Degree on either ſide, beginning from O or N, and taking OS or

NQ for the South Part, and OR or PN for the North Part, because NO has been taken for the Eaſtern Part.

Plate 14. Fig. 43.

Thus will the Plain of the Dial be, by the Circles of Longitude and Latitude, divided into ſeveral little Squares, as a Map of the whole Earth is, in which you may write down what Places of the World you will, according to their Longitudes and Latitudes, as they are plac'd in the general Map. Then you may know the Hour in any propos'd Place of the Earth, which will be mark'd in the Dial where the Circles of Longitude of that Place will be cut by the Shadow of the *Axis* of the Dial, for the Curve Hour-line, which goes thro' that Point of Section, will ſhew the Hour requir'd.

PROBLEM XX.

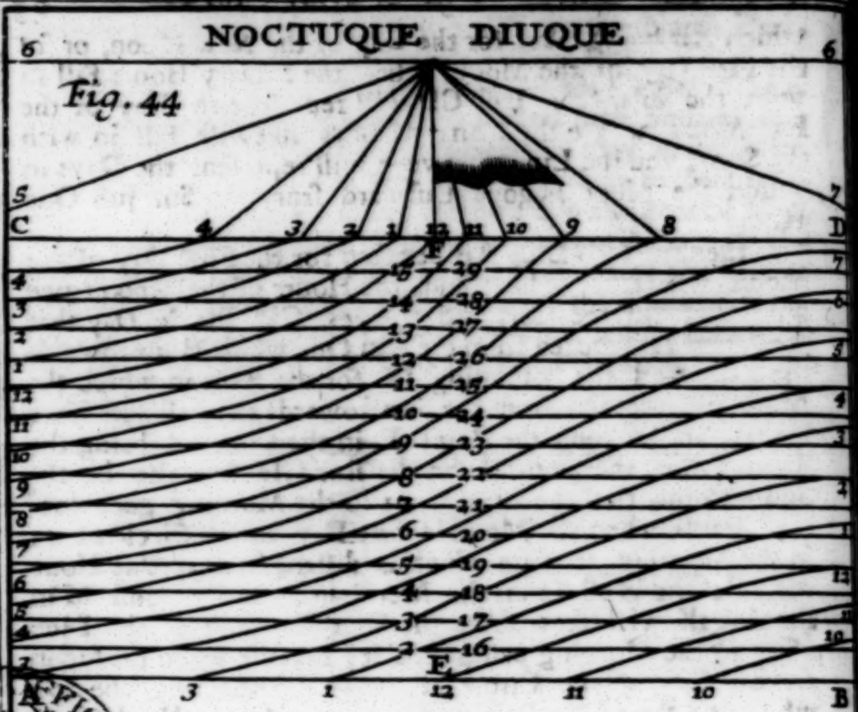
How to draw an Horizontal Moon-Dial.

A *Moon-Dial* is ſuch a Dial as ſhews by Night, by the Beams of the Moon, what Hour of the Sun it is. The Conſtruction of this Dial is founded upon the ſame Principle as the foregoing, and all the Difficulty depends upon the Moon's proper Motion, by which it goes from the Sun ſo far Eaſtward every Day, as to riſe about 3 Quarters of an Hour later each Day, than the Day before: So that when the Moon is New, or in Conjunction with the Sun, if it cou'd Enlighten the Earth, the next Day, or its ſecond Day, it wou'd riſe 3 Quarters of an Hour later, its third Day an Hour and a half, and ſo on, till it comes to be Full Moon, and conſequently 12 Hours or 180 Degrees diſtant from the Sun, at which time it will ſhew the ſame Hours as the Sun; becauſe when the Sun ſets that Day, the Moon, which is Diametrically oppoſite to it, riſes about the ſame time, and ſucceeds in its Room.

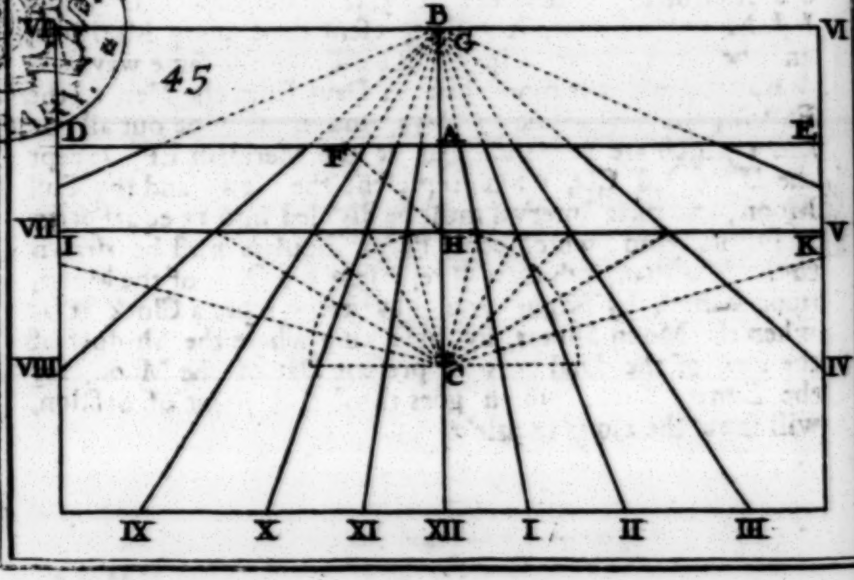
So that if you know the Moon's Age, you may eaſily know by Night what a Clock it is by the Beams of the Moon upon an Horizontal Dial, *viz.* by adding to the Hour which the Moon ſhews, as many times 3 Quarters of an Hour as Days are paſs'd ſince the New Moon. But as this Addition is troubleſome, you may draw the Lunary Hours in an Horizontal Dial, and in any other Dial, with the Days of the Moon's Age, after a Method like the foregoing.

Having deſcrib'd an Horizontal Dial as uſual, draw as before, the Two Lines AB, CD, perpendicular to the Meridian EF, and divide their Interval EF into 12 equal Parts, to draw thro' the Points of Diviſion other Lines perpendicular to the Meridian, or parallel to the Two Firſt AB, CD, the Firſt of which

Fig. 44


NOCTUQUE DIUQUE
C
D
F
E
A
B

45


B
G
VI
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VIII
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IX
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XI
XII
I
II
III

which AB being taken for the Day of the New Moon, or for the Firſt Day of the Moon, where the Lunary Hours fall in with the Solar, the Laſt CD will repreſent the Day of the Full Moon, where the Lunary Hours likewiſe fall in with the Solar, and the Lines between will repreſent the Days in which the Moon is gone Eaſtward from the Sun juſt One Hour. *Plate* 14. *Fig.* 43.

If then the Firſt Line AB be taken for the Firſt Day of the Moon, where, as we have ſaid, the Hours of the Moon agree with thoſe of the Sun, and the next GH for the Day that the Moon has receded from the Sun One whole Hour towards the Eaſt; and the following IK for the Day in which the Moon has receded from the Sun towards the Eaſt juſt Two Hours, and ſo on; the Point E in the Line AB being the South Point, the point L in the line GH will alſo be the South Point, tho' the Line of 11 in the Morning goes thro' it; becauſe when the Moon is that Day in the Circle of 11 in the Morning, the Sun which is diſtant from it One Hour towards the Weſt is in the Meridian: and the point M in the line IK will likewiſe be the South Point, tho' the Line of 10 in the Morning paſſes thro' it; becauſe when the Moon is that Day at the Circle of 10 in the Morning, the Sun which is diſtant from it Two Hours towards the Weſt, is in the Meridian. Wherefore if you draw thro' the points E,L,M, *&c.* a Curve, it will repreſent the Lunary Meridian, and the other Lunary Lines will be drawn the ſame way.

But becauſe it is more than 12 Days from the New to the Full Moon, *viz.* about 15 Days, you muſt wipe out all the Lines which are perpendicular to the Meridian EF, except the Two AB, CD, which repreſent the New, and the Full Moon, and their Interval muſt be divided into 15 equal parts, in Points, thro' which other Perpendiculars muſt be drawn to the Meridian, which will repreſent the Days of the Moon, upon which by Night you may know what a Clock it is when the Moon Shines, by obſerving where the Shadow of the *Axis* of the Dial cuts the preſent Day of the Moon, for the Lunary Line, which goes thro' that point of Section, will ſhew the Hour requir'd. *Plate* 15. *Fig.* 44.

CHAPTER III.

Of Erect, *or* Vertical Dials.

SUCH a Dial as is drawn upon an Erect, or Vertical Plain, is call'd an *Erect* or *Vertical Dial*. It may be *Direct*, or *Regular*, when it is drawn upon the Surface of a Vertical Plain which looks directly towards One of the Four Cardinal Points of the World; and *Declining*, or *Irregular*, when it is drawn upon the Surface of a Declining Plain. An *Erect Dial* is call'd a *South Dial*, when it is drawn upon the Surface of a Vertical Plain, which being parallel to the Firſt Vertical, directly faces the South: And *North Dial*, when the Surface of the Plain on which it is drawn directly faces the North. But it is call'd a *Meridian Dial*, when it is drawn upon the Surface of a Plain parallel to the Meridian: and *Eaſt Meridian*, when the Surface ſtands full Eaſt; and *Weſt Meridian*, when it ſtands full Weſt. We ſhall in the following Problems, give the Deſcription of all theſe Dials in their Order.

PROBLEM I.

How to deſcribe an Erect South Dial.

Plate 15. *Fig.* 45. HAving thro' the Foot of the Style A drawn the Plumb-line BC, which will be the Meridian, draw to it thro' the ſame Foot of the Style A, the Perpendicular DE, which will be the Horizontal-line, where the Hour-lines muſt be terminated when the Dial has but One Style, becauſe the end of the Shadow will never go above the Horizontal-line.

Set off the Length of the Style upon the Horizontal DE, at AF, and at the Point F, (which will be the Dividing Center of the Meridian BC) above the Horizontal DE, draw the Angle AFB of the Height of the Pole, and below it the Angle AFH of the Complement of the Height of the Pole, to have upon the Meridian, at B, the Center of the Dial, and at H the Point of the Equinoctial IK, which, as in the Horizontal Dial, is drawn perpendicular to the Meridian BC, and divided after the ſame manner into Hours, *viz.* by ſetting off the *Radius* of the Equator HF, upon the Meridian BC, from H to the Point C, which will be the Dividing Center of the Equinoctial IK, *&c.*

SCHO-

SCHOLIUM.

You ſee by this fundamental Conſtruction, that this Dial is only an Horizontal Dial drawn for a Latitude equal to the Complement of the Height of the Pole above the Horizon, as at *Paris*, for the Latitude of 41 Degrees, becauſe the Pole there is 49 Degrees high: Likewiſe the Elevation of the Pole above the Horizon of the Plain, which is the Prime Vertical, is equal to the Complement of the Height of the Pole above the Horizon of the Place where you are. Wherefore all that we have ſaid concerning the Horizontal Dial in the foregoing Chapter, may be applied to this, conceiving the Pole to be rais'd above the Horrizon, the Complement of the Latitude of the Place where you are. *Plate* 15. *Fig.* 45.

We have plac'd the Center of the Dial B above the Horizontal DE, becauſe it repreſents the lower Pole, which is fac'd by the Plain of the Dial; for if the Sun was in that Pole, which here is the Antarctick Pole, and its Rays cou'd penetrate the Earth, and after that manner enlighten the Plain, the Shadow of the End of the Style wou'd fall above the Horizontal-line and come to that Center B.

You may alſo find the Hour-points upon the Horizontal-line DE, dividing it, as the Horizon of the Place is divided, by the Horary Circles, according to the Table of the Horary Arches: *viz.* by ſetting off the Length of the Style upon the Meridian, from A the Foot of the Style to the Point G, which will be the dividing Center of the Horizontal DE, and where conſequently you muſt with the Meridian BC make Angles equal to the Horary Arches found in the Table, *page* 23, over-againſt the Height of the Pole above the Horizon; or what is the ſame thing, by applying at the point G, the Center of an Horizontal Dial drawn for the Latitude of the place, in ſuch manner that its Meridian may fall in with the Meridian BC, and then the other Hour-lines being produc'd with a Thread or a Ruler will upon the Horizontal-line DE give the Horary Points requir'd.

Upon Walls theſe ſort of Dials are uſually drawn before the Style is fix'd, that there being nothing over the Foot of it, the Work may be perform'd more eaſily. But as it is hard to fix the Style very exactly ſo as it may remain ſtreight without ſpoiling any thing, I ſhou'd rather chuſe to fix it before the Dial was begun, and inſtead of ſetting it at Right-angles have it at oblique Angles, when you wou'd have no Axis, (fixing it into the Wall as well as one can) that its Foot may be free, which you will find, by deſcribing at pleaſure from the End of the Style, an Arch upon the Plain of the

Plate 15. *Fig.* 45. the Dial, and chuſing where you will upon that Arch the Three moſt diſtant Points that you can, to find by means of thoſe Three Points the Center of the Arch, which will be the Foot of the Style, whoſe Length is the diſtance from that Center to the End of the Style.

What we have ſaid hitherto ſuppoſes the Sphere to be Oblique, but if the Sphere is parallel, in ſuch manner that the Pole is 90 Degrees above the Horizon, the Horizontal-line DE will repreſent the Equinoctial, and the Dial will be a *Polar Dial*, whoſe Conſtruction we have taught *Probl.* 6. *Chap.*2. and if the Sphere be Right: ſo that the Two Poles of the World be in the Horizon, the Foot of the Style A will repreſent One of the Two Poles of the World, and the Horizontal-line DE the Six a Clock-line, ſo that the Dial will be an Equinoctial Dial, whoſe Deſcription we have taught *Probl.* 7. *Chap.* 2.

We have in the Figure only repreſented the Hours from Six in the Morning to Six at Night, becauſe the Sun ſhines no longer upon ſuch a Plain, the other Surface of it towards the North, being enlighten'd the other Hours of the Day, when the Day is more than 12 Hours long, as at *Paris* in the longeſt Days of Summer, from Sun-riſe till Eight in the Morning, and from Four in the Afternoon till Sun-ſet. Here follows the manner of drawing ſuch a Dial.

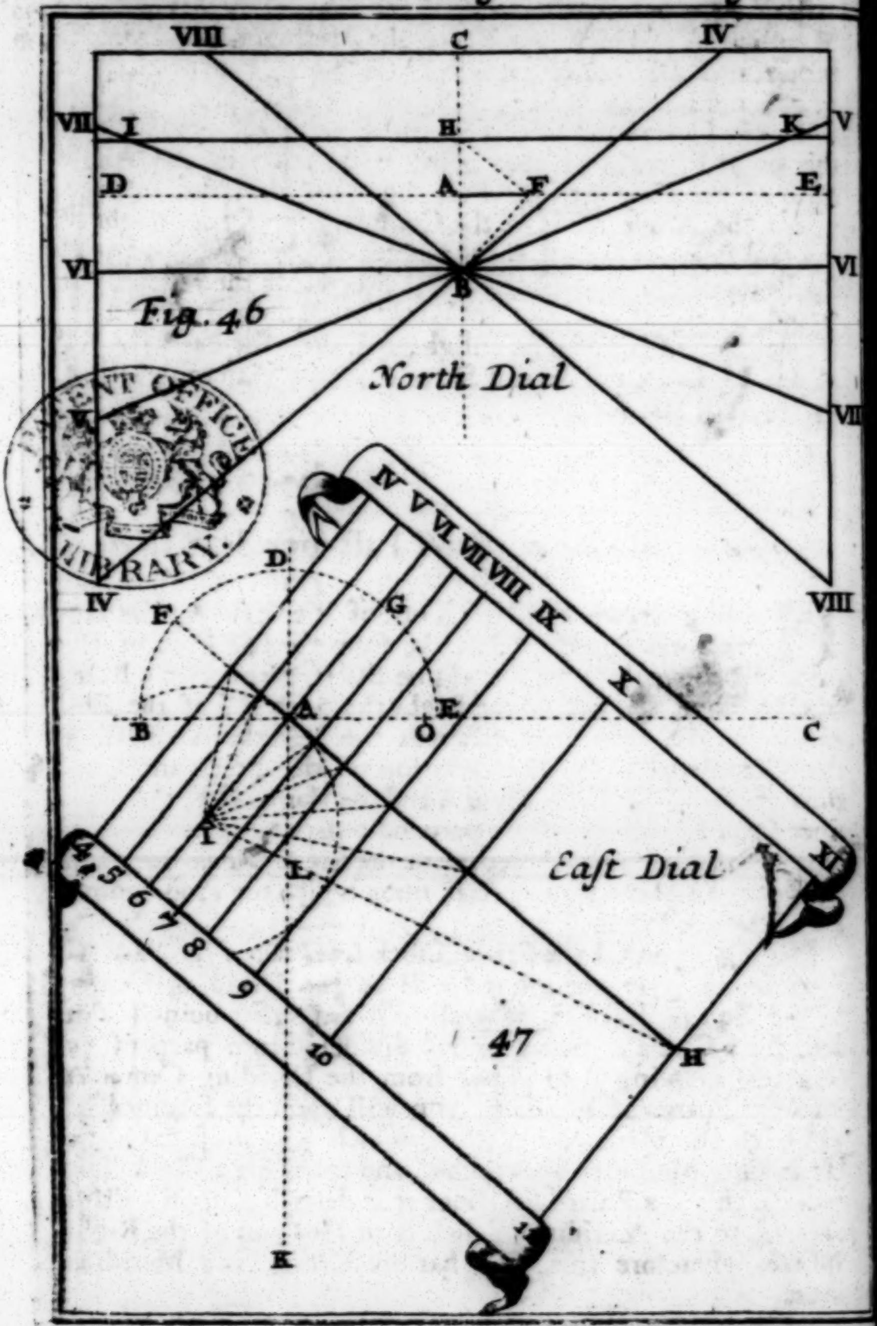
PROBLEM II.

How to deſcribe an Erect North-Dial.

AN Erect *North-Dial* is drawn the ſame way as the South-Dial, for it is evident that the Dials which are drawn upon the Two Oppoſite Faces of the ſame Plain are the ſame kind of Dials, only with this difference, *viz.* that the Order of the Points and Lines of the One, is contrary to that of the other; ſo that what is on the Right in one Superficies, is on the Left in the other; and what is at Top in One, is at Bottom in the Other.

Thus becauſe the Center of the Dial is above the Horizontal-line in the Erect South-Dial, it muſt be below it in this, becauſe this Center repreſents the upper Pole, which the Plain faces, in which if the Sun was, the Shadow of the Style wou'd tend downwards; and likewiſe becauſe the Equinoctial Line is below the Horizontal in the Erect South-Dial, it muſt be above it in this, which ſhews that this Dial is the ſame as the foregoing turn'd upſide-down, but we muſt come to the Practice.

Ha-


VIII
C
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VI
Fig. 46
North Dial
V
VII
IV
VIII
IV
V
VI
VII
VIII
IX
X
XI
D
G
F
B
A
E
O
C
I
L
East Dial
4
5
6
7
8
9
10
11
47
H
K

Plate 16. *Fig.* 46.

Having as before, drawn thro' the Foot of the Style A, the Meridian BC, which repreſents the Midnight-line, and the Horizontal DE, which repreſents the North Part of the Horizon, and having, as before, ſet off the Length of the Style upon the Horizontal DE, from the Foot of the Style A, to the point F, make at that point F downwards the Angle AFB of the Height of the Pole above the Horizon, and upwards the Angle AFH of the Complement of the Height of the ſaid Pole, to have at B the Center of the Dial, and at H the Point of the Equinoctial IK; afterwards the reſt will be ended as before; but you need only mark upon it the Hours at which the Plain can be enlighten'd, *viz.* for *Paris* from 4 in the Morning till 8, and from 4 in the Afternoon till 8, that is, till Sun-ſet.

PROBLEM III.

How to deſcribe an Erect Eaſt-Meridian-Dial.

Fig. 47.

HAving drawn thro' the Foot of the Style A, the Horizontal-line BC, deſcribe from the ſaid Foot of the Style A, above the Horizontal-line BG the Semi-circle BDE, to take upon it towards the Right the Arch EG of the Elevation of the Pole, and towards the Left the Arch BF of the Complement of the Elevation of the Pole, and draw thro' the ſaid Foot of the Style A and the Points G, F, the Two lines GI, FH, which will be perpendicular to one another, of which the Firſt GI will repreſent the Six a Clock-line, and the Second FH the Equinoctial, upon which the Hour-points may be mark'd thus:

Having upon GI the Six a Clock-line, taken the Part AI equal to the Style, the point I will be the Dividing Center of the Equinoctial FH, wherefore if from that point I you deſcribe a Circle at pleaſure, and divide it into parts of 15 Degrees, drawing Right-lines from the Dividing Center A thro' the Points of Diviſion, you will upon the Equinoctial FH have the Hour-points, thro' which you muſt draw the Hour-lines parallel to one another, and to the Six a Clock-line, becauſe this is a Polar Dial, ſince it is deſcrib'd upon a Plain parallel to the Meridian, which is an Horizon of the Right Sphere, wherefore this Dial has no Center, nor Meridian Line.

SCHOLIUM.

Inſtead of marking the Hour-points upon the Equinoctial FH, they might have been mark'd upon the Horizontal BC, draw-

Plate 16. *Fig.* 47. drawing to it thro' the Foot of the Style A, the Perpendicular AL equal to the Style, to have at L the Dividing Center of the Horizontal BC, where you muſt apply the Center of an Horizontal Dial, in ſuch manner that its Six a Clock-line may fall in exactly with the Perpendicular AL, for thus the other Hour-lines being produc'd till they meet the Horizontal-line BC, you will at each of thoſe Points of Interſection have the Hour-points requir'd.

You may alſo mark the Hour-points upon the Firſt Vertical DK, which is drawn Perpendicularly thro' the Foot of the Style A, *viz.* by taking from the Foot of the Style A, upon the Horizontal BC, the part AO equal to the Length of the Style, to have at O the Dividing Center of the line DK, where you muſt apply the Center of a Vertical or Erect South-Dial, in ſuch manner that its Six a Clock-line may fall in with the part BO; for thus its Hour-lines being produc'd will upon the line DK give the Hour-points requir'd.

PROBLEM IV.

How to draw an Erect Weſt-Meridian Dial.

Plate 17. *Fig.* 49. HAving drawn, as before, thro' the Foot of the Style A, the Horizontal BC, deſcribe from the ſaid Foot of the Style A, above the Horizontal BC, the Semi-circle BDE, to take upon it towards the Left the Arch EG of the Height of the Pole, and towards the Right the Arch BF of the Complement of the Height of the Pole; and draw thro' the ſame Foot of the Style A, and the points G, F, the Six a Clock-line GI, and the Equinoctial FH, then make an end of the reſt as before.

SCHOLIUM.

It is evident, that in this Dial and in the foregoing, when the Pole is exactly 90 Degrees above the Horizon, the Six a Clock-line will be perpendicular to the Horizontal, which in that caſe will repreſent the Equinoctial: And when the Pole is not above the Horizon, the Six a Clock-line will be the ſame as the Horizontal, and conſequently the Equinoctial the ſame as the Vertical.

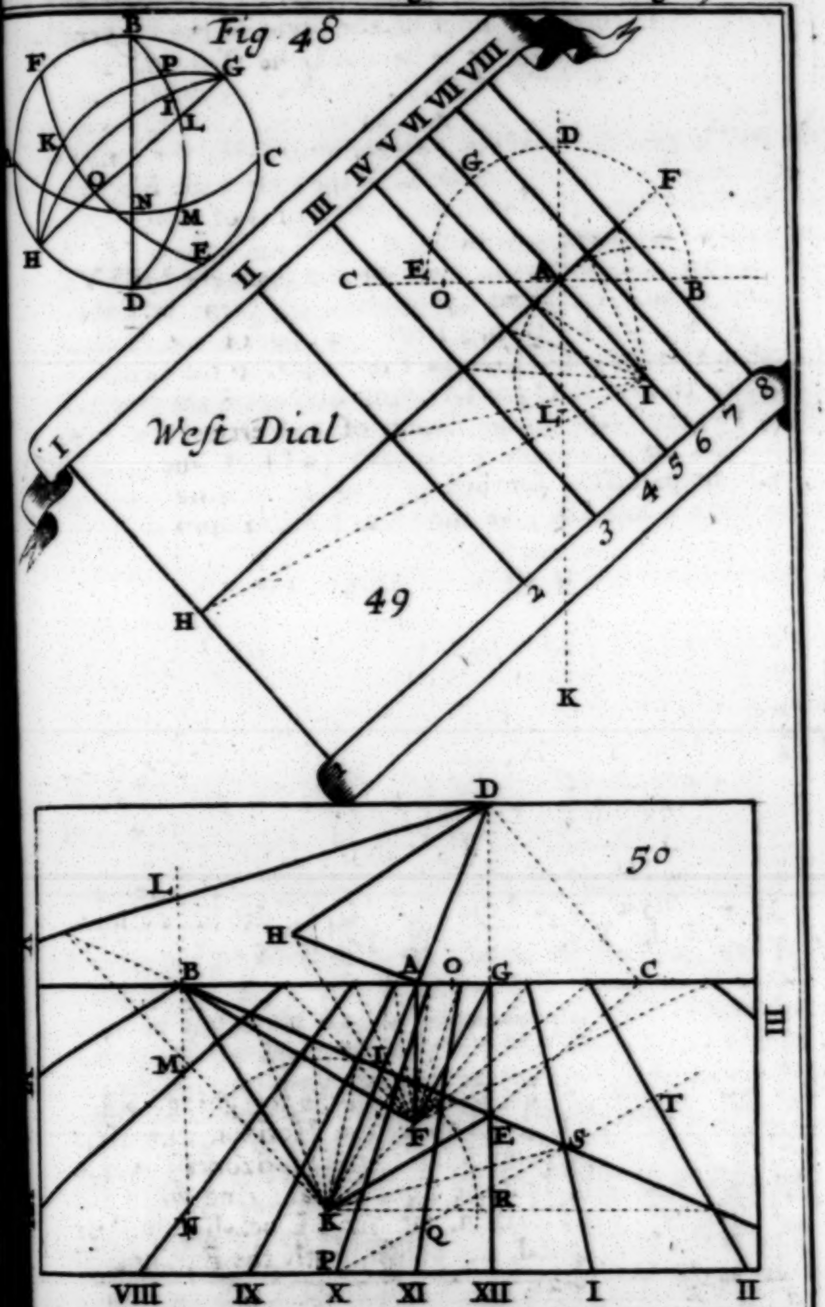
Fig 48
B
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P
G
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L
K
C
O
N
M
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D
West Dial
I
II
III
IIII
V
VI
VII
VIII
D
G
F
C
E
O
A
B
I
L
2
3
4
5
6
7
8
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PROBLEM V.

How to deſcribe an Erect Dial *declining from the South.*

Having thro' the Foot of the Style A, drawn the Horizontal-line BC, by *Lem. 7. Chap. 1.* and the Meridian DE being drawn upon the Plain, by *Lem. 16. Chap. 1.* draw thro' the Foot of the Style A, the Line AF, equal to the length of the Style, and perpendicular to the Horizontal BC, and draw the Line of Declination FG, whoſe length being ſet off upon the Horizontal-line BC, from G to C, which will be the dividing Center of the Meridian DE; then at that Point C, make upwards the Angle ACD, of the Height of the Pole, and downwards the Angle ACE of the Complement of the Height of the Pole, to have upon the Meridian the Center of the Dial at D, and at E, a Point of the Equinoctial. *Plate 17. Fig. 50.*

To draw that Equinoctial, you muſt find another Point upon the Horizontal-line BC, drawing thro' the Point F, which is the Dividing Center of the Horizontal, to the Line of Declination FG, the Perpendicular FB, which will upon the Horizontal give the Six a Clock Point B, thro' which alſo the Equinoctial paſſes; if then thro' the Two Points B, E, the Right-line BE, be drawn, it will be the Equinoctial; which may alſo be found another way, when B the Six a Clock-point cannot be mark'd upon the Horizontal BC, which may happen when the Plain declines but a little.

Having drawn thro' D the Center of the Dial, and A the Foot of the Style, the Subſtylar DK, draw to it thro' the Foot of the Style A, the Perpendicular AH, equal to the Length of the Style, to have at H the Dividing Center of the Subſtylar, and draw the *Axis* of the Dial DH, to which you muſt draw thro' the Point H, the Perpendicular HI, which will be the *Radius* of the Equator, and will upon the Subſtylar DK, give I the Point of the Equinoctial, which will be drawn thro' that Point I perpendicular to the Subſtylar, as BE; upon which you muſt mark the Hour-points in the following manner.

Having ſet off HI the *Radius* of the Equator upon the Subſtylar, from I to K, which will be the Center of the Equator; that is, the Dividing Center of the Equinoctial, join the Right-lines KE, or KB, or only KE, if you have not B the Six a Clock-point, and deſcribe at pleaſure from the Point K a Circle, which you muſt divide into Parts of 15 Degrees, beginning

Plate 17. beginning from the Line KE or KB, to draw from the ſaid *Fig.* 50. Point K, thro' the Points of Diviſion Right-lines, which being produc'd, will, upon the Equinoctial BE, give the Hour-points, thro' which you muſt from the Center D, draw the Hour-lines, and the Dial will be finiſh'd.

By this Method one can only mark the Hours at which the Plain is Shin'd upon at the Equinoctial Times; and to have all the Hours at which it may be enlighten'd at any time, you muſt mark the Hour-points upon the Horizontal-line BC, by applying to its Dividing Center F, the Center of an *Horizontal Dial*, in ſuch manner, that the Meridian may fall in with the Line of Declination FG, or its Six a Clock Line with the Line FB; for if with a Thread or a Ruler you produce the other Hour-lines, you will upon the Horizontal BC, have the Horary Points requir'd.

When you have the Six a Clock Point upon the Horizontal-line, as B, you may draw thro' that Point B, the Perpendicular LN, which will repreſent the Prime Vertical, and which may be alſo divided into Hours, by applying to its Dividing Center O, (which is found by making BO equal to BE) the Center of an Erect *South Dial*, in ſuch manner, that the Six a Clock-line may fall in with the Horizontal BC, for then the other Hour-lines being produc'd, will, upon the Vertical LN, give the Hour-points requir'd.

Thus you ſee the Reaſon of the uſual Manner of drawing the Hour-lines, which are beyond the Six a Clock-line, when their Points are too far diſtant upon the Horizontal, as it happens here to the Five a Clock-line; One Point of which, as L, is found upon the Vertical LN, by ſetting off the Diſtance BM, from the Point B to the Seven a Clock-line at BL, to have at L the Five a Clock-point.

If in making uſe of the Equinoctial, one cannot mark upon it all the Hours beyond XII, (which may alſo happen to the Horizontal, when the Declination of the Plain is very great) make uſe of the Hour-lines which are drawn on the other ſide of the Style, as far as the Six a Clock-line (which in this Suppoſition is always on the Dial, becauſe its Point B will not be very far from the Foot of the Style A) finding by their means the Hour-points which you want upon any Line parallel to the Six a Clock-line. *For Example*, Upon the Line PT, (which cuts the Meridian at R,) the Line of 11 a Clock at Q, the Line of 10 at P, *&c.* For if you ſet off the ſpace RQ at RS, you will have at S the One a Clock-point, which is as far diſtant from 12, as the Line of 11, and likewiſe if you ſet off the diſtance RP at RT, you will have at T the Line of 2, which is as far diſtant from 12 as the Line of 10, and ſo on.

DEMONSTRATION.

Becauſe the Six a Clock Circle is perpendicular to the Meridian Circle, if you ſuppoſe a Plain which paſſing thro' the line PT, is parallel to the Six a Clock Circle, that Plain will alſo be perpendicular to the Meridian Circle, and parallel to the Axis of the World. Thus wou'd the Dial drawn upon ſuch a Plain be a Polar Dial, ſo that its Hour-lines wou'd be parallel to One another and to the Meridian of the Plain, which wou'd go thro' the point E, and be the common Section of that Plain; and of the Meridian which is perpendicular to the Plain: Wherefore in this Polar Dial the Hour-lines are equally diſtant on either ſide of the Subſtylar, and alſo the Horary Points mark'd upon the line PT are on either ſide equally diſtant from R the South-point. *Plate* 17. *Fig.* 50.

SCHOLIUM.

The Angle ADH, which is the Angle of the Axis, with the Subſtylar ſhews the Height of the Pole above the Plain, and the Angle IKE, which meaſures the Arch of the Equator comprehended between the Subſtylar and the Meridian, (that is, between the Meridian of the Plain and the Meridian of the Place) ſhews the difference of Longitude in reſpect to the Horizon of the place, and the Horizon of the Plain. Wherefore if you know thoſe Angles, (by meaſuring them with a Protractor, or rather by Spherical Trigonometry, as we ſhall teach hereafter,) you will know in the Map what Countries of the Earth have the Plain of the Dial for their Horizon, that is, have their Horizon parallel to the Plain of the Dial.

Let ABCD be the Meridian, AC the Horizon, BD the the Prime Vertical, which going thro' the Zenith B, and Nadir D, cuts the Horizon at Right-angles at the point N, in ſuch manner that the Spherical Angle CND, or BNC may be a Right. Let the Horizon of the Plain be the Vertical BLD, in ſuch manner that the Spherical Angle MBN may be the Declination of the Plain, which will be meaſur'd by the Arch of the Horizon MN, whoſe Complement CM meaſures the Spherical Angle MBC, which will conſequently be the Complement of the Declination of the Plain. Let the Meridian of the Plain be the Circle GIH, which going thro' G, H, the Two Poles of the World, cuts the Horizon of the Plain at Right-angles at the point I, and the Equator EF at the point K, in ſuch manner that the Angle of the *Axis* with the Subſtylar, or the Height of the Pole above the Plain *Fig.* 48.

Plate 17. *Fig.* 50. Plain may be the Arch GI, and the difference of Longitudes the Arch FK, or Spherical Angle BGI. Laſtly, let the Six a Clock Circle be GLH, which cuts the Horizon of the Plain, at Oblique Angles, at the point L, and the Equator EF at Right-angles, at the point O.

Firſt to find the Height of the Pole above the Plain, or the Arch IG, you muſt conſider that in the Spherical Triangle BIG, Rectangular at I, the Oblique Angle GBI, or the Complement of the Declination of the Plain is known, and the Hypotenuſe BG, or the Complement of the Height of the Pole above the Horizon. Wherefore the Side GI may be found by this Analogy,

As the Whole Sine :
To the Sine of the Complement of the Height of the Pole : :
So the Sine of the Complement of the Declination of the Plain :
To the Sine of the Height of the Pole above the Plain.

In the ſame Triangle BIG, the Angle BGI: or the difference of Longitudes may be found by this Analogy,

As the Whole Sine :
To the Sine of the Height of the Pole above the Horizon : :
So the Tangent of the Complement of the Declination of the Plain :
To the Tangent of the Complement of the difference of Longitudes.

Fig. 48. If you wou'd find the Arch IB, (which is equal to the Angle which the Subſtylar makes with the Meridian, as may be known by deſcribing from the Center of the Dial taken for the Center of the Earth, a Circle which will repreſent the Horizon of the Plain, and whoſe Arch comprehended between the Subſtylar and Meridian, (which is the ſame as the Arch IB) meaſures the Angle of the Subſtylar with the Meridian,) you may do it in the ſame Rectangular Triangle BIG, by this Analogy,

As the Whole Sine :
To the Sine of the Declination of the Plain : :
So the Tangent of the Complement of the Height of the Pole :
To the Tangent of the Angle of the Subſtylar with the Meridian,

which is the ſame Angle as the Equinoctial makes with the Horizontal-line : Becauſe in this Dial theſe Two are perpendicular to one another, ſince they repreſent Circles which are perpendicular to one another ; and One of thoſe Circles is perpen-

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Gnomonicks or Dialling Plate 18 Pag. 95

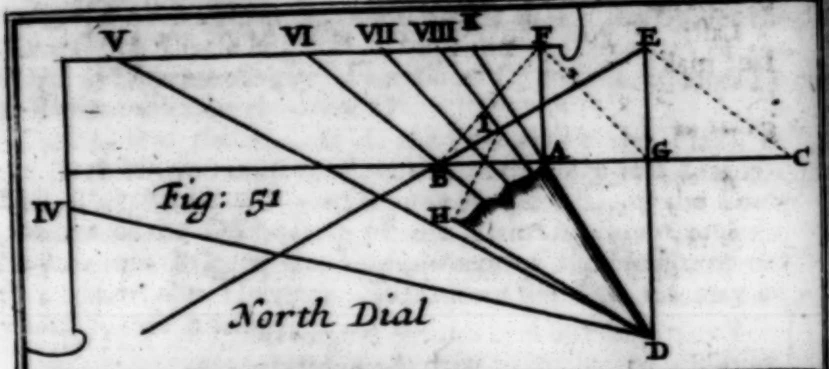
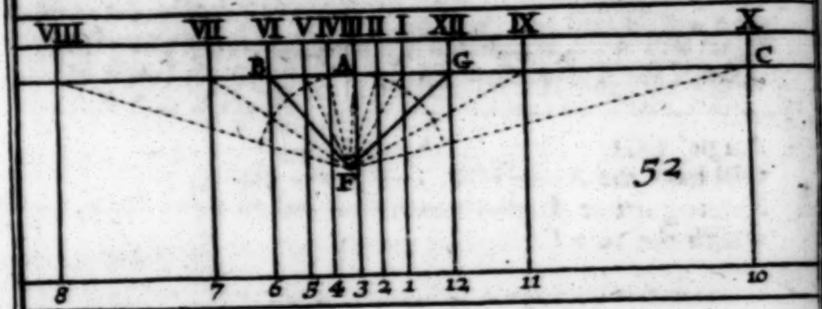


Fig: 51

North Dial



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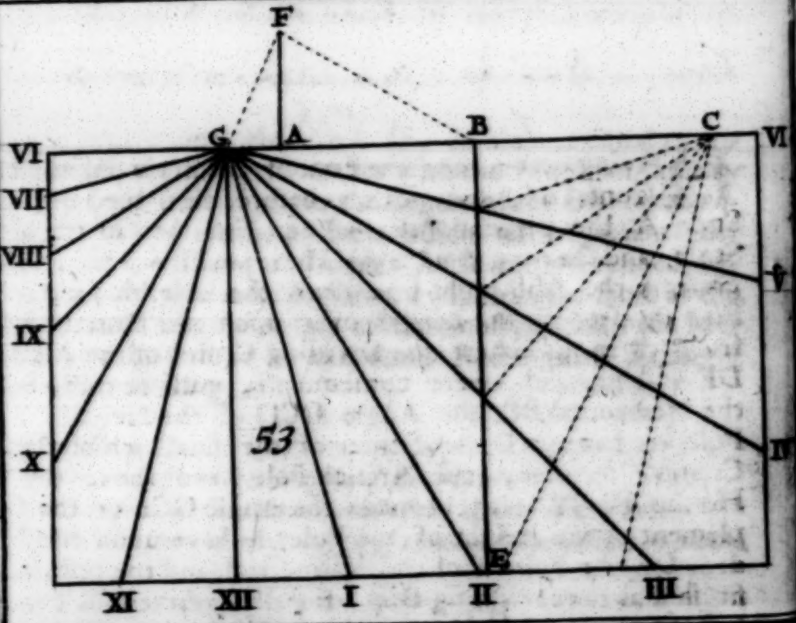

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perpendicular to the Plain of the Dial, *viz.* the Meridian of the Plain. *Plate* 17. *Fig.* 48.

Laſtly, if you wou'd find out what Angle the Six a Clock-line makes with the Meridian, or the Arch BL; you may do it by this Analogy, in the Spherical Triangle BGL, Rectangular at G,

As the Whole Sine:
To the Sine of the Declination of the Plain::
So the Tangent of the Height of the Pole:
To the Tangent of the Complement of the Angle requir'd.

You might alſo find out the Angles of the other Hour-lines with the Meridian, or with the Subſtylar: As, *for Example*, if you wou'd find out the Angle of the 10 a Clock-line with the Subſtylar, ſuppoſing the 10 a Clock-circle to be GPH, in which caſe the Horary-diſtance, or the Spherical-angle BGP will be of 30 Degrees, which being here taken from the Angle BGI, which is the difference of Longitude, you will have the Angle IGP, and in the Spherical Triangle PIG Rectangular at I, you may know the Side PI, or the Angle which the 10 a Clock-line makes with the Subſtylar.

PROBLEM VI.

How to draw an Erect Dial *declining from the North.*

HAving drawn thro' the Foot of the Style A, the Horizontal BC, draw to it, as before, thro' the Foot of the Style A, the perpendicular AF equal to the Length of the Style, and having found the Meridian DE, which here repreſents the Mid-night-line, draw the Line of Declination FG, and ſet off the Length of it upon the Horizontal BC, from G to C, where the Dividing Center of the Meridian DE will be, and where conſequently muſt be made below the Horizontal BC, the Angle GCD of the Height of the Pole, to have at D the Center of the Dial, which in this Country repreſents the Arctick Pole; and above the ſaid Horizontal BC muſt be made the Angle GCE of the Complement of the Height of the Pole, to have upon the Meridian DE, the Point E of the Equinoctial, and the reſt muſt be finiſh'd as the foregoing Dial; for all we have ſaid concerning it will ſerve for this, which is only the ſame Dial upſide-down. *Plate* 18. *Fig.* 51.

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Plate 18. Fig. 52. What we have ſaid concerning theſe Two Laſt Dials, ſuppoſes the Sphere to be Oblique; but if it was Parallel, the Dial wou'd have no Center, and conſequently wou'd be a *Polar Declining*, becauſe the Horizon of the Plain wou'd be an Horizon of the Right Sphere; wherefore, in this caſe, the Horizontal-line BC will repreſent the Equinoctial, which muſt conſequently be divided into Hours, by means of an *Equinoctial Dial*, or of a Circle divided into Parts of 15 Degrees, always applying its Center to the Point F, the Dividing Center of the Horizontal, in ſuch manner, that the Meridian may fall in with the Line of Declination FG, *&c.*

Fig. 53. But if the Sphere be a Right Sphere, the Dial will be an *Equinoctial Declining*, ſo that the Horizontal will be the Six a Clock-line, and the Point G of the Meridian will be the Center of the Dial, which is thus deſcrib'd:

Having to the Line of Declination FG, drawn the Perpendicular FB, to have upon the Horizontal GC the Point B of the Equinoctial, and having drawn thro' that Point B the Perpendicular BE, which will be the Equinoctial, ſet off the Hypotenuſe BF on BC, to have at C the Dividing Center of the Equinoctial, which muſt be divided into parts of 15 Degrees, as uſual, *&c.*

Tho' this be call'd an *Equinoctial Dial*, it does not follow that the Horary Angles are all equal to one another, for they have the ſame inequality, in reſpect to the Six a Clock-line GC, as the Horary Angles of a Horizontal Dial, in reſpect of the Meridian, for a Latitude equal to the Complement of the Declination of the Plain; as may be known, if you conſider the Horizontal GC as the Meridian of an Horizontal Dial, whoſe Center is G, and Equinoctial BE, *&c.*

Therefore the Angles, which the Hour-lines make with the Meridian at G, the Center of the Dial, muſt be known by this Analogy,

As the Whole Sine:
To the Sine of the Complement of the Declination of the Plain::
So the Tangent of the Horary Diſtance:
To the Tangent of the Complement of the Angle requir'd.

From what has been ſaid it is plain, that the Angles which the Hour-lines make with the Six a Clock-line, or Horizontal GC, are equal to the Horary Arches for a Height of the Pole equal

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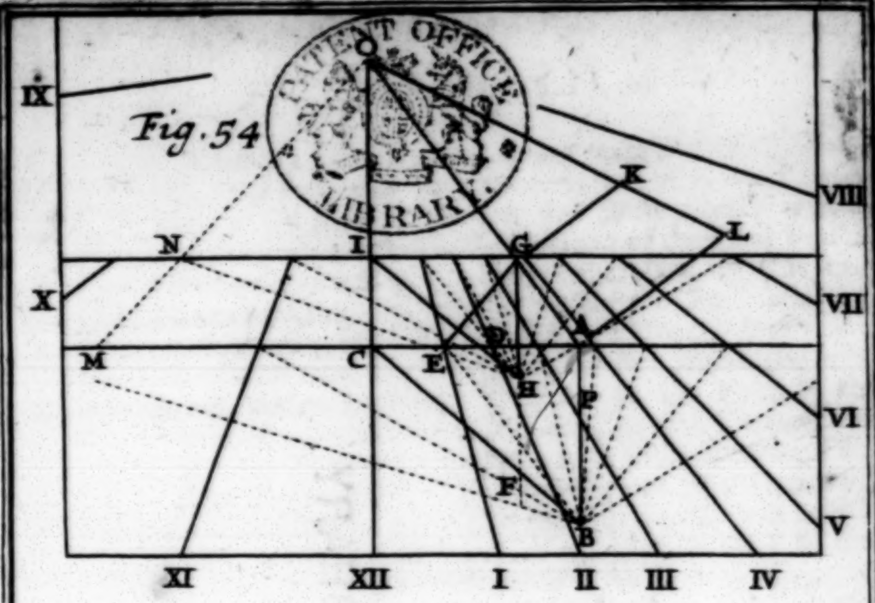
Fig. 54



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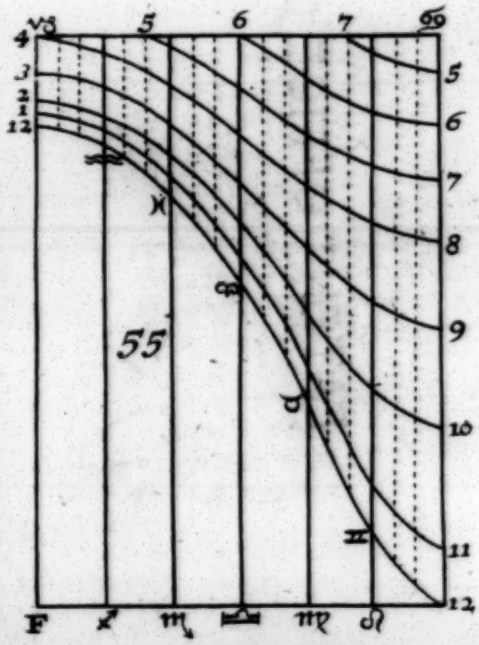

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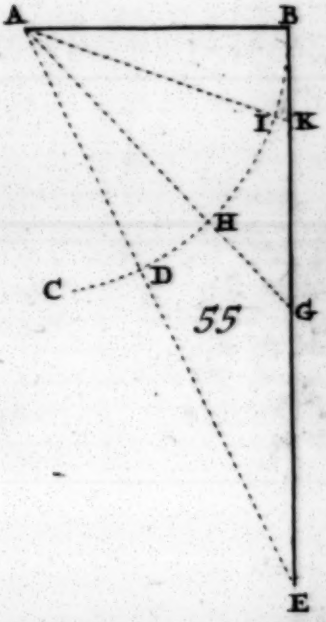

equal to the Complement of the Declination of the Plain; and therefore that the Table of the Horary Arches for every Degree of the Elevation of the Pole (*Page* 23.) may ſerve to deſcribe this Dial, which is no ſooner ſeen but underſtood. *Plate* 18. *Fig.* 53.

PROBLEM VII.

How to draw an Erect Declining Dial *without a Center.*

WHEN the Declination of the Plain is very great, ſo that the Meridian cannot eaſily be drawn upon the Plain, and conſequently the Center of the Dial cannot be had, or when the Height of the Pole above the Horizon is very great, which may likewiſe hinder one from having the Center of the Dial at a convenient Diſtance for drawing the Dial; in ſuch a caſe, the Dial may be drawn without a Center, by means of Two Horizontal-lines, thus.

Having mark'd the Horary Points upon the Horizontal MA, by the application of an Horizontal Dial (its Center being at the Point B the Dividing Center of the Horizontal MA, and its Meridian upon the Line of Declination BC) draw at pleaſure the Perpendicular GF, cutting the Horizontal MA at the point D, and the Line of Declination BC, at the point F, and draw DE equal to BF, to make at the point E the Angle DEG, equal to the Height of the Pole, by means of the Line EG, which will upon the Perpendicular GF give the Point G, which muſt be conſider'd as a ſecond Foot of the Style, thro' which, and thro' the firſt Foot of the Style A, muſt be drawn the Subſtylar GA. *Plate* 19. *Fig.* 54.

Draw thro' the ſame Point G, to the Horizontal MA, the Parallel NG. which will be a ſecond Horizontal-line, which muſt be divided into Hours, by taking GH, equal to DF, and drawing to the Line of Declination BC, thro' the Point H, the Parallel HI, which will be a ſecond Line of Declination, upon which conſequently muſt be apply'd the Meridian of an Horizontal Dial, whoſe Center muſt be at the Point H: For thus you will have Two points of each Hour-line, which will be enough to finiſh the Dial. If thro' the Point A, be drawn the Line AL, perpendicular to the Subſtylar GA, and equal to the firſt Style AB, and thro' the Point G, the Line GK, perpendicular to the ſaid Subſtylar GA, and equal to the ſecond Style GH, the Line KL will be part of the *Axis* of the Dial.

DEMONSTRATION.

Plate 19. *Fig.* 54. Let us ſuppoſe the Line of Declination BC to meet the Horizontal MA at the Point C, which wou'd be the South Point, in reſpect of the Style AB, and if you had no other Horizontal but MA, you ſhou'd draw from that South Point C, the Meridian OC perpendicular to the Horizontal MA, and the Dividing Center M of that Meridian OC wou'd be found, by making CM equal to the Hypotenuſe BC, and if at M the Angle CMO was made of the Height of the Pole, you wou'd upon the ſaid Meridian OC have the Center of the Dial at the point O, thro' which, and the Foot of the Style A, ſhou'd be drawn the Subſtylar OA.

If you ſhou'd draw a Second Horizontal NG, parallel to the Firſt MA, the point G, where it wou'd meet the Subſtylar OA, wou'd be the Foot of a Second Style for that Second Horizontal NG, and the point N wou'd be the Dividing Center of the Meridian OC: And to determine the Length of that Second Style, you ſhou'd draw thro' the point G, to the Horizontal NG the Perpendicular GH, which muſt be terminated at H, by drawing thro' the point I a Second Line of Declination IH, parallel to the Firſt BC, and then the Line GH wou'd be the Length of the Second Style, and the Line NI wou'd be equal to the Hypotenuſe HI, as the Line CM is equal to the Hypotenuſe BC.

If the Style GH be produc'd towards F, and the Line GF be taken for a Second Meridian, and the Point G for a Second Center of the Dial, you will find the Dividing Center of that Second Meridian, by drawing from the Center G the Line GE parallel to the Line OM; that the Angle DEG may be equal to the Angle CMO, and conſequently to the Height of the Pole, and the point E will be the Dividing Center of the Meridian GF, in reſpect of which, the Foot of the Style will be A, and its Length AP will be determin'd by the Third Line of Declination DP, parallel to the Firſt BC, or to the Second HL. Then it will be eaſily known, that ſince the point E is the Dividing Center of the Second Meridian GF, the Line DE is equal to the Hypotenuſe DP, that is BF; which agrees with the foregoing *Conſtruction*, where we have made GH equal to DF; for it muſt be ſuch, becauſe if to the Two equal Lines FH, DG, be added in *that Figure* the common Line DH, you will have DF equal to GH.

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The ſame Operation will ſerve, when the Angle of Declination ABC is a Right, or of 90 Degrees: in which caſe the Dial will have no Meridian, being a *Polar South Dial*, which conſequently may be deſcrib'd by the Precepts of *Probl.* 3, *or* 4. *Plate* 19. *Fig.* 54.

PROBLEM VIII.

How to draw a Cylindrick Dial.

WE call *Cylindrick Dial* a Vertical Dial, which is drawn upon the Surface of a Cylinder, by means of the Heights of the Sun above the Horizon, as you will find 'em in the following Table, for every 10 Degrees of each Sign of the Zodiack, at each Hour of the Day, for the Latitude of *Paris*, (which we have every where ſuppos'd of 49 Degrees, becauſe it is very near the matter) for which the Dial of *Fig.* 55. has been drawn in this manner.

A Table of the Height of the Sun above the Horizon, at every Hour of the Day, calculated for 49 Degrees Latitude.

H	XII.	XI.	X.	IX	VIII.	VII.	VI.	V.
S.	D.M	D.M.	D.M	D.M.	DM.	D.M.	DM.	D.M.
30.♋	64.30	61.56	55.19	46.36	37. 1	27.12	17.12	8. 22
20.10	64. 9	61.33	55. 1	46.18	36.44	26.56	17.32	8. 4
10.20	63. 2	60.31	54. 4	45.28	35.59	26. 8	16.22	7. 12
♊ ♌	61.12	58.49	52.34	44. 7	34.40	24.51	15. 7	5. 50
20.10	58.48	56.30	50.29	42.14	32.54	23. 7	13.21	3. 57
10 20	55.52	53 42	47 57	39 55	30.42	20.58	11.12	1. 40
♉ ♍	52.30	50.30	45. 1	37.14	28.10	18.29	8. 40	
20.10	48.51	46.58	41.44	34.13	25.19	15.43	5. 54	
10.20	44 58	43 12	38.15	31. 0	22.18	12.48	2. 59	
♈ ♎	41. 0	39 20	34.37	27.38	19. 9	9 .47		
20.10	37. 2	35.26	30.58	24.15	15.58	6 .42		
10.20	33. 9	31.40	27.24	20.55	12.51	3 .44		
♓ ♏	29.30	28. 4	23.58	17.42	9. 50	0. 54		
20.10	26. 8	24.46	20.51	14.45	7. 5			
10 20	23.12	21.52	18. 5	12.12	4. 42			
♒ ♐	20 48	19.30	15.48	10. 3	2.41			
20.10	18.48	17 44	14. 6	8.27	1.12			
10.20	17.51	16.38	13. 3	7.27	0.18			
♑.30	17.30	15.15	12 42	7 .8				
H.	XII.	I.	II.	III.	IV.	V.	VI.	VII.

Plate 19. Fig. 55. Draw upon a Sheet of Paper the Rectangular Parallelogram F12♋♑, whoſe Breadth F12, or ♋♑, muſt be equal to (or a ſmall matter leſs than) the Circumference of the propos'd Cylinder, and the Length F♑ equal to (or ſomething leſs than) the Height of the ſaid Cylinder.

Having made this *Preparation*, draw upon another Paper the Line BE equal to the Line ♋12, and draw to it, thro' the Point B, the Indefinite Perpendicular AB, which will be terminated at the Point A by the Line EA, which muſt with the

Plate 19. *Fig.* 55.

the Line BE make the Angle BEA of 26 Degrees and 30 Minutes; that is, of the Complement of the Height of the Sun at Noon, when it is in ♋, that is, in the Summer Tropick, and that Perpendicular AB, thus terminated, will repreſent the Length of the Style, by means of which, and of the foregoing Table, the Dial muſt be thus drawn:

Divide the Breadth ♋♑, which repreſents the upper part of the Cylinder, into 6 equal parts, and draw thro' the Points of Diviſion Lines parallel to the Length F♑, which will repreſent the Beginnings of the Signs of the Zodiack, upon which you muſt mark the Hour-points by means of the Table of the Sun's Height, thus:

For Example, If you wou'd find the point of 9 in the Morning, or of 3 in the Afternoon, upon the line ♑12, which repreſents the Tropick of ♑, you muſt from the point A draw the Arch BC, to take upon it the Arch BH of 46 Degrees and 36 Minutes, ſuch as the Height of the Sun above the Horizon is at 3 in the Afternoon, or 9 in the Morning, and having drawn the Right-line AHG, ſet off the Length of the Line BC upon the Tropick of ♋, from the point ♋, to the point 9, which will be the point requir'd.

Likewiſe if you wou'd find upon the ſame Tropick ♋12, the 6 a Clock Point, take the Arch BI of 17 Degrees and 32 Minutes, ſuch as you find in the foregoing Table to be the Sun's Height at 6 a Clock, when the Sun is in the beginning of ♋, and having drawn the Right-line AIC, ſet off the Length of the Line BK upon the ſaid Tropick of ♋, from the point ♋ to the point 6, which will be the point requir'd, and ſo on.

After this manner muſt the Hour-points be mark'd upon the other Parallels of the Signs, and even upon the Spaces between, dividing each Interval into Three equal parts, each of which will repreſent 10 Degrees, that you may draw the Hour-lines more exactly, joining the Points which belong to the ſame Hour by Curves, and the Dial will be finiſh'd, which you muſt glue neatly to the Surface of the Cylinder, ſo that the Line ♑♋ may be exactly parallel to the Horizon, when the Cylinder is perpendicular to the Horizon, as it muſt be when you wou'd know the Hours, by means of the Rays of the Sun, which muſt be done by fixing the Style to the Point of the Sign in which the Sun is upon the Line ♑♋, and turning the Cylinder (which muſt hang freely, by means of a Ring) till the Shadow of the Style covers the Parallel of the Sun, and then the End of the ſaid Shadow will ſhew the Hour requir'd.

SCHOLIUM.

Plate 19. *Fig.* 55. A Dial may alſo be drawn upon the Surface of a Cylinder parallel to the Axis of the World, by dividing the Circumference of One of its Two Baſes, which in that Situation will be Two Circles, parallel to the Equator into 24 equal Parts, drawing thro' the Points of Diviſion the Hour-lines parallel to one another, and perpendicular to the Oppoſite Baſes, upon each of which may be drawn an Equinoctial Dial, and upon each Dial you will know the Hours (on the Upper from the Vernal to the Autumnal Equinox, and on the Under Surface from the Autumnal to the Vernal Equinox) by means of an *Axis* rais'd at Right-angles in the Center.

As for the Cylinder, you may know the Hours upon it without a Style, as upon the Surface of a Globe, whoſe *Axis* has the ſame Situation as the *Axis* of the World; and ſuch a Dial is call'd a *Natural Dial*. Or if you wou'd have a Style, you may have it as long as you will, turning it about the Cylinder directly towards the Sun, in ſuch manner that it may caſt no Shadow on its Sides, as upon a Globe, *&c.*

PROBLEM IX.

How to draw an Exact Pocket-Dial *upon a Quadrant.*

WE call a *Pocket-Dial* ſuch a One as you may carry about with you, to know the Hours upon it at any time, by turning the Style towards the Sun, as in the Foregoing Cylindrick Dial, and in the Three following, which we ſhall make in the Quarter of a Circle as ABC, by means of the foregoing Table of the Heights of the Sun above the Horizon, in the following manner.

Plate 20. *Fig.* 56. Having taken BC, the Quarter of the Circumference of a Circle whoſe Center is A, for the Tropick of ♋, and having made the Part A4 equal to about One Third of the *Radius* AB, deſcribe from the Center A thro' the Point 4, another Quarter of a Circle (which you muſt look upon as the Tropick of ♑) and divide the other part 4B into 6 equal parts in points, thro' which you muſt deſcribe from the ſaid Center A, other Quadrants, which will repreſent the Parallels of the other Signs of the Zodiack, upon which the Hour-points muſt be mark'd in the following manner.

To find (*for Example*) the Point of 12 upon the Parallel which is common to the beginning of the Two Signs ♊, ♌, where

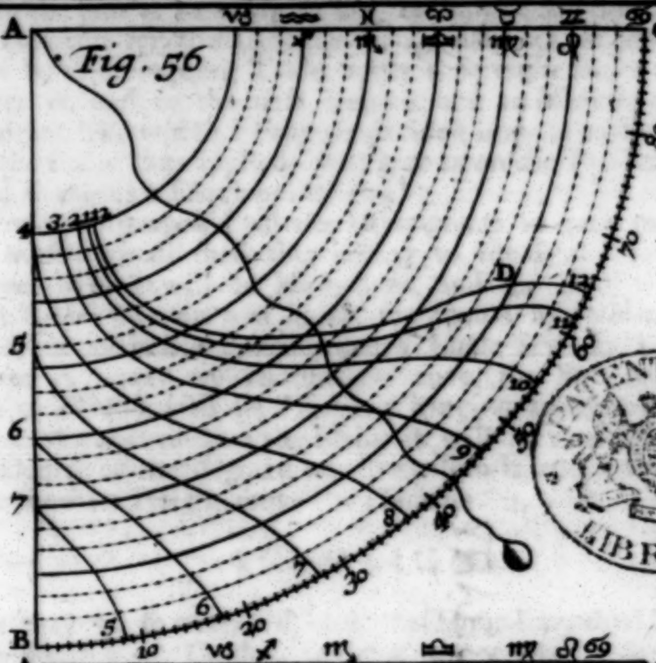


Fig. 56
A
C
B
D
♑ ♒ ♓ ♈ ♉ ♊ ♋
♐ ♏ ♎ ♍ ♌
3 2 1 12
4
5
6
7
5 6 7 8 9 10 11 12
10 20 30 40 50 60 70 80

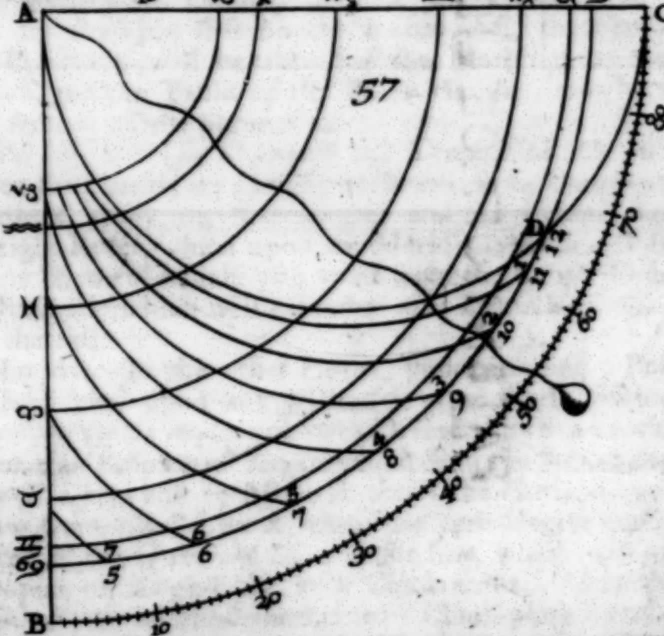

57
A
C
B
D
♑ ♐ ♏ ♎ ♍ ♌ ♋
♑ ♒ ♓ ♈ ♉ ♊ ♋
12 11 10 9 8 7 6 5
1 2 3 4 5 6 7
10 20 30 40 50 60 70 80

Plate 20. *Fig.* 56.

where the Sun at 12 being 61 Degrees and 12 Minutes above the Horizon for the Latitude of 49 Degrees, as you may know by the foregoing Table, apply a Streight Ruler to the Center A, and to the 61ſt Degree, and 12 Minutes of the Quadrant BC, which I ſuppoſe divided into its 90 Degrees, and the Ruler thus applied, will give upon the Parallel propos'd D the 12 a Clock-point, *&c.*

Having thus mark'd all the Hour-points in each parallel, you muſt joyn all thoſe that belong to the ſame Hour by Curves, which will be Hour-lines, and the Dial will be finiſh'd, where you will know the Hours by holding the Quadrant in ſuch a Poſition that a Little Style fix'd to the Center A, may by its Shadow cover the line AC, and then a Thread with its Plummet hanging freely from the Center A, as it touches the Plain, it will ſhew the Hour in the Degree of that Sign in which the Sun is, and at the ſame time ſhew its Height above the Horizon.

SCHOLIUM.

Fig. 57.

It may be ſo contriv'd, that the Hour-lines ſhall be Circumferences of Circles, without any conſiderable Error, *viz.* by drawing firſt on the *Radius* AC, the Semi-circle ADC, which will be taken for the Meridian, by means of which, and the Table of the Sun's Heights, may be drawn the Arches of the Signs, thus:

To deſcribe (*for Example*) the Tropick of ♋, in which when the Sun is, its Height at Noon is 64 Degrees and 30 Minutes, apply on that Degree and the Center A, a very Streight Ruler, which upon the Meridian ADC will give the point D, thro' which you muſt from the Center D deſcribe a Quadrant which will repreſent the Tropick of ♋, and ſo of the reſt.

To deſcribe the other Hours, you muſt find 3 Points of each of 'em, upon any 3 Parallels after the following manner: If, *for Example*, you wou'd find the 10 a Clock Point upon the Tropick of ♋, in which Sign the Sun at that time is 55 Degrees and 19 Minutes above the Horizon, you muſt draw from the Center A, thro' the 55th Degree and 19 Minutes of the Quadrant BC a Right-line, which will upon the Tropick of ♋ give the 10 a Clock-point. After the ſame manner you will find another 10 a Clock-point upon another Parallel, *for Example*, upon the Equator, and a Third upon the Tropick of ♑, and if an Arch be drawn thro' thoſe Three Points, it will repreſent the Line of 10, and 2 a Clock.

Plate 21. *Fig.* 58.

The Hours may be alſo repreſented by Right-lines, without any conſiderable Error, taking firſt the Quadrant BC for the

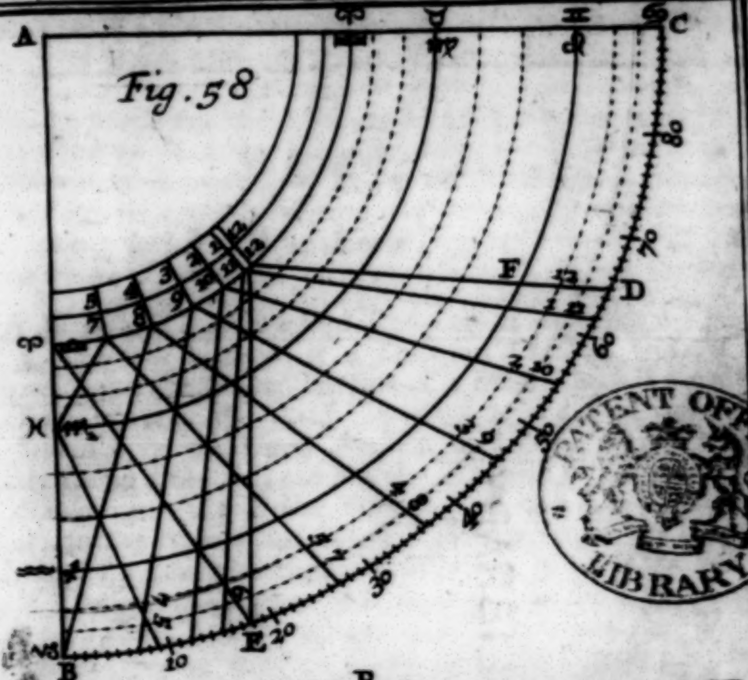
Plate 21. *Fig.* 58. the Two Tropicks of ♋ and ♑, and the Quadrant which we have drawn thro' the middle of the Two *Radii* AB, AC, for the Equator, after which you muſt upon each of thoſe Circles find One Point of each Hour, to joyn the Two Points which belong to the ſame Hour, after this manner.

For Example, If you wou'd find the Point of 12 upon the Equator, in which Circle the Sun at 12, is 41 Degrees high; draw from the Center A thro' the 41ſt Degree of the Quadrant BC a Right-line, which will upon the Equator give the Point of 12; thro' which and thro' the point D of 64 Degrees and 30 Minutes of the ſaid Quadrant BC, conſider'd as the Tropick of ♋, drawing a Right-line 12 D, it will repreſent the Meridian, which will ſerve for the Six Northern Signs, *viz.* from the Vernal to the Autumnal Equinox, becauſe the Sun at Noon is 64 Degrees and 30 Minutes high, when it is in the Tropick of ♋: and if thro' the ſame Point 12 and the point E of the 17th Degree and 30 Minutes of the ſame Quadrant, conſider'd as the Tropick of ♑, you draw the Right-line 12 E, it will repreſent the Meridian for the Six Southern-Signs, that is, from the Autumnal to the Vernal Equinox, becauſe the Sun at Noon is 17 Degrees and 30 Minutes high; when it is in the Tropick of ♑, as you may know by the foregoing Table.

After the ſame manner muſt the other Hour-lines be mark'd, as well for the Northern as for the Southern Signs, from the Twelve a Clock to the Six a Clock-line, which paſſes thro' the middle of the *Radius* AB: And to have the others, you may after the ſame manner find One point of 'em upon any one of the Parallels of the other Signs, after we have deſcrib'd 'em thus:

To deſcribe (*for Example*) the Parallel which ſerves for the beginning of the Two Signs ♊, ♌, which ſhall alſo ſerve for the beginning of Two other Signs as far diſtant from the Equator, or the Two Tropicks, *viz.* ♒, ♐, knowing by help of the foregoing Table, that the Sun being in the beginning of ♊ and of ♌, which are Two Northern Signs, is at Noon 61 Degrees and 12 Minutes above the Horizon, you muſt draw from the Center A thro' 61 Degrees and 12 Minutes of the Quadrant BC, a Right-line, which upon 12 D, the Meridian of the Northern Signs, will give the point F, thro' which you muſt from the Center A draw the Fourth Part of a Circle, which will repreſent the Parallel requir'd, *&c.*

Fig. 58


A
C
D
F
E
B

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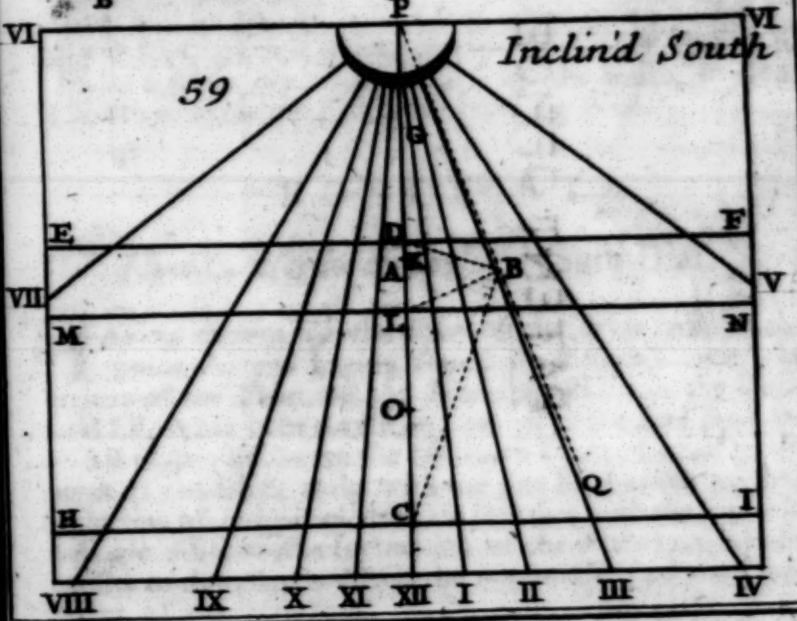
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VI
VI
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K
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VIII
IX
X
XI
XII
I
II
III
IV

CHAP. IV.

Of Inclin'd Dials.

AN *Inclin'd Dial* is a Dial drawn upon an Inclin'd Plain, and we ſhall call it a *South Inclin'd Dial*, when it is drawn upon the Upper Surface of a Plain which is directly inclin'd towards the South: A *North Inclin'd Dial*, when it is drawn upon the Upper Surface of a Plain which is directly inclin'd towards the North, in which caſe that which is drawn upon the Under Surface will be a South Inclin'd Dial: an *Eaſt Inclin'd Dial*, when it is drawn upon the Upper Surface of a Plain which inclines directly towards the Eaſt: A *Weſt Inclin'd Dial*, when it is drawn upon the Upper Surface of a Plain, which inclines directly towards the Weſt, in which caſe that which is drawn upon the Oppoſite Under Surface will be an *Eaſt Inclin'd Dial*: And laſtly, an *Inclin'd Declining Dial*, when it is drawn upon the Upper Surface of an Inclin'd Plain which declines from the South, or from the North towards the Eaſt, or towards the Weſt.

PROBLEM I.

How to deſcribe an Inclin'd South-Dial.

Plate 21. *Fig.* 59.

HAving determin'd the Foot of the Style AB, at the point A, and having found the Zenith C, with the Vertical of the Plain, or the Meridian PC, and the Horizontal EF as has been taught in *Chap.* 1. at the End B of the line AB perpendicular to the Meridian PC, and equal to the Length of the Style, make with the line BC (which we ſhall call the line of *Inclination*, becauſe it makes with the line AB, the Angle ABC of the Inclination of the Plain) on the ſide Oppoſite to that Angle ABC, the Angle CBQ of the Complement of the Height of the Pole, or of the diſtance of the Zenith from the Pole, by the Right-line BQ, which being produc'd, meets here the Meridian PC at the Center of the Dial P, which in this Figure happening to be above the Horizontal, ſhews that the Upper Surface of the Plain faces

Plate 21. *Fig.* 56. faces the Lower or Depreſs'd Pole, and conſequently that the Inclination of the Plain is greater than the Height of the Pole.

Having thus found the Center of the Dial at the point P, which in this Example repreſents the Depreſs'd Pole, what remains is to find out the Hour-points upon the Horizontal EF, or upon the Vertical HI, (which is drawn thro' the Zenith C, perpendicular to the Meridian PC, or upon the Equinoctial MN, which is alſo drawn perpendicular to the Meridian PC thro' the point L of that Meridian) which are found by drawing thro' the point B to the *Axis* PQ, the Perpendicular BL, which will be the *Radius* of the Equator, whoſe Length being ſet off upon the Meridian at LO, you will have at O the Center of the Equator, from which you may divide the Equinoctial NM into parts of 15 Degrees, or into Hours by means of an Equinoctial Dial, whoſe Center muſt be applied at the Dividing Center O; or according to the ſhort Methods which we have taught in the Deſcription of the Horizontal Dial, *Probl.* 1, *Chap.* 3.

If you wou'd find all the Hour-points upon the Horizontal EF, ſet off the Length of the Hypotenuſe DB, from D upon the Meridian to the point G, which will be the Dividing Center of the Horizontal, where you muſt apply the Center of an Horizontal Dial, *&c.* Likewiſe if you wou'd mark the Hour-points upon the Vertical HI, ſet off the Line of Inclination BC, from the Zenith C upon the Meridian at the point K, which will be the Dividing Center of the Vertical, where you muſt apply the Center of a Vertical South Dial, *&c.*

SCHOLIUM.

The Center of the Dial P happen'd here to be above the Horizontal-line EF, becauſe the Inclination of the Plain is greater than the Height of the Pole: For if it ſhou'd be leſs, that Center wou'd be below it and above the Zenith C: And if the Inclination ſhou'd be equal to the Height of the Pole, this Center wou'd be neither above nor below; that is, the Dial wou'd have no Center, and therefore it wou'd be a Polar Dial, and the Equinoctial wou'd go thro' the Foot of the Style A, becauſe in that Caſe the line BQ wou'd be parallel to the Meridian BC.

It is plain, that when this Dial has a Center, it may be look'd upon as a Horizontal Dial, drawn for ſuch an Elevation of the Pole, as is equal to the difference between the Height of the Pole of the Horizon of the Place where you are, and the Inclination of the Plain; becauſe in this Figure if from the Angle of Inclination ABC, you take away the Angle LBC, which is equal to the Latitude of the place, there

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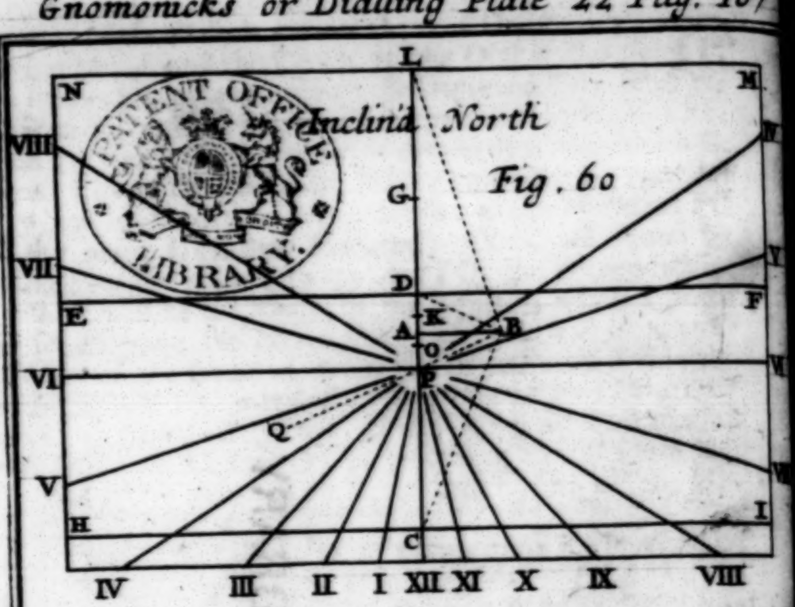
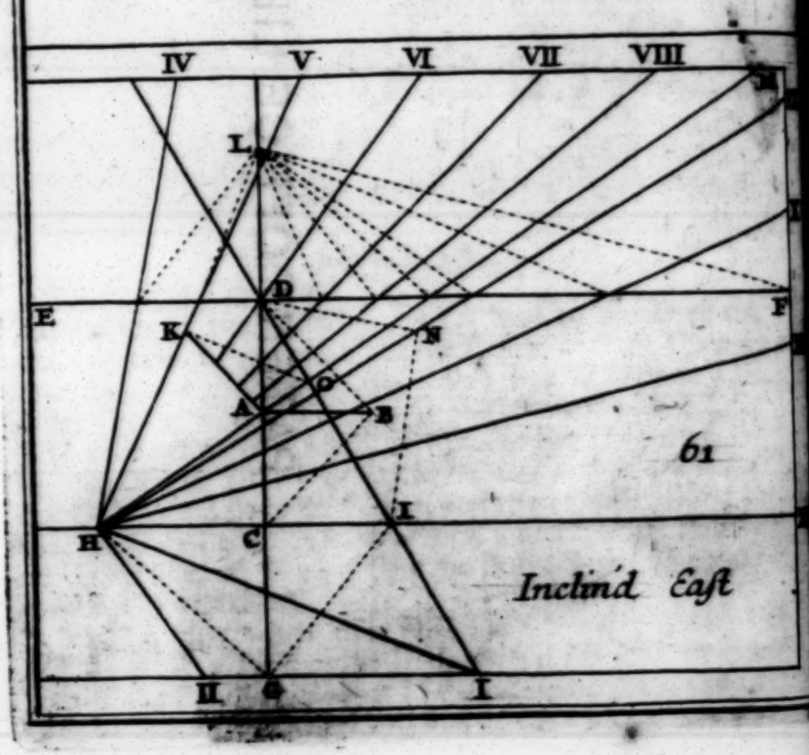


Inclin'd North

Fig. 60



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Inclin'd East

there will remain the Angle ALB, which is equal to the Angle ABP of the *Axis* with the Subſtylar, or to the Height of the Pole above the Plain, for which Elevation you may eaſily draw the Dial by means of the Table of Horary Arches.

Plate 21. *Fig.* 59.

PROBLEM II.

How to draw a North-Inclin'd-Dial.

Plate 22. *Fig.* 60.

A *North-Inclin'd-Dial* is deſcrib'd after the ſame manner as a South, ſo that its Conſtruction is taught in the foregoing Problem, only inſtead of drawing the Angle CBQ on that ſide which is oppoſite to the Angle of Inclination ABC, you muſt draw it towards that Angle as you ſee in the Figure) to have the Center of the Dial, P, which will repreſent the Arctick Pole, and which conſequently will be below the Horizontal EF, becauſe that Pole is above our Horizon. It happens here to be below the Foot of the Style A, becauſe the Inclination of the Plain is greater than the Complement of the Height of the Pole; that is, the Angle ABC is greater than the Angle CBQ: For if the Inclination of the Plain was leſs than the Complement of the Height of the Pole above the Horizon, that Center wou'd be between the Foot of the Style A, and the point D of the Horizontal, and it wou'd be exactly at the Foot of the Style A, if the Inclination of the Plain was equal to the Complement of the Height of the Pole; and in ſuch a Caſe the Dial wou'd be an Equinoctial Dial.

SCHOLIUM.

It is plain, that to find in this Dial the Height of the Pole above the Plain, or the Angle of the *Axis* with the Subſtylar, that is, the Angle APB, you muſt only add the Complement of the Height of the Pole above the Horizon to the Complement of the Inclination of the Plain, when that Inclination is greater than the Complement of the Pole above the Horizon: And when the Inclination of that Plain is leſs than the Complement of the Height of the Pole above the Horizon, you muſt add together the Inclination of the Plain and the Height of the Pole above the Horizon, to have the Height of the Pole above the Plain, by means of which, and the Table of Horary Arches, it will be eaſy to draw this Dial.

In this, and the foregoing Problem, I have taken no notice of the Dial drawn on the inner Surface of the Plain, becauſe it is deſcrib'd the ſame way, only in an inverted order, as we we have already ſaid. The Upper *Polar Dial* ſhews the Hours from 6 in the Morning 'till 6 at Night, and the Under *Polar Dial* ſhews the other Hours in Summer. The Upper Equinoctial ſhews all the Hours of the Day in this Country, whilſt the Sun is in the Northern Signs, *viz.* from the Vernal to the Autumnal Equinox, and the Lower ſhews the Hours from 6 in the Morning to 6 at Night, from the Autumnal to the Vernal Equinox.

PROBLEM III.

How to draw an Inclin'd East-Dial.

Plate 22. *Fig.* 61. HAVING determin'd the Foot of the Style AB, at the point A, and having found as before, the Zenith C, the Horizontal-line EF, and the Vertical of the Plain LG, which in this Plain will Repreſent the Prime Vertical, draw thro' the Zenith C the Meridian HI, parallel to the Horizontal EF, or perpendicular to the Vertical LG, to which you muſt draw thro' the Foot of the Style A, the Perpendicular AB, equal to the Length of the Style, to have at B the Dividing Center of that Vertical, upon which, conſequently you muſt mark the Hour-points, by means of an Erect *South Dial*, applying its Center at the point B, and its Meridian upon the line of Inclination BC, or its Six a Clock-line, upon the Line BD, becauſe the point D is the Six a Clock Point.

You may alſo find the Hour-points upon the Horizontal EF. ſetting off the length of the Line BD upon the Vertical LG, from D to L, which will be the Dividing Center of the Horizontal EF, where you muſt apply the Center of an *Horizontal Dial*, in ſuch manner, that the Six a Clock-line may fall in with the Vertical LG, and then the other Hour-lines being produc'd, will cut the Horizontal EF at the Hour-points requir'd, thro' which you muſt from H, the Center of the Dial, draw the Hour-lines, which Center you will find upon the Meridian HI, by ſetting off the Line of Inclination BC upon the Vertical at CG, and making at the point G towards the Left, the Angle CGH of the Complement of the Height of the Pole.

If you wou'd have the Equinoctial-line, draw thro' the Foot of the Style A, and the Center of the Dial H the Subſtylar HM, to which you muſt, thro' the Six a Clock-point D, draw the Perpendicular DI, which will be the Equinoctial: Or elſe at the Point G, make the Angle GCI, of the Height of the Pole above the Horizon, to have upon the Meridian

ridian HI the Point I, thro' which, and the Point D, you muſt draw the Equinoctial DI: Or elſe draw thro' the Foot of the Style A the Line AK, equal to the Length of the Style AB, and perpendicular to the Subſtylar HM, and having drawn the Axis HK, draw to it, thro' the Point K, the Perpendicular KO, which will upon the Subſtylar give the Point O, thro' which you muſt draw a Perpendicular to it, which will paſs thro' the Point D, and will be the Equinoctial; upon which you may, if you will, mark the Hours, ſetting off the *Radius* of the Equator OK, upon the Subſtylar, from O to the Point N, which will be the Center of the Equator, where you muſt apply the Center of an *Equinoctial Dial*, in ſuch manner, that its Meridian may fall in with the Line NI, or its Six a Clock-line with the Line ND, *&c*,

Plate 22. *Fig.* 66.

SCHOLIUM.

This Conſtruction plainly ſhews, That this Dial is the ſame as an *Erect Declining Dial*, drawn for the Complement of the Height of the Pole above the Horizon of the Place where you are, taking the Vertical LG of this Dial for the Horizontal of the *Erect Decliner*, and the Angle of Inclination ABC of that Dial, for the Angle of the Declination of the *Erect Decliner*, ſo that the Horizontal EF of that Dial muſt be taken for the Vertical-line of the *Erect Declining Dial*. Thus what we have ſaid in *Probl.* 5. *Chap* 3. concerning the *Erect Declining Dial*, will ſerve for this.

PROBLEM IV.

How to draw a Weſt-Inclin'd Dial.

A *Weſt-Inclin'd Dial* is drawn after the ſame manner as an *Eaſt-Inclin'd Dial*, with this difference only, *viz.* That the Center of the Dial muſt be mark'd on the Right: Wherefore it wou'd be needleſs to ſay any thing more about it, or to give a particular Figure, which wou'd be the ſame as the 61ſt Figure ſeen thro' the Paper on the backſide, if it be held up to the Light, or by holding it to a Looking-Glaſs, where the Reflection of the *Eaſt-Inclin'd Dial* will be the Figure of the *Weſt-Inclin'd Dial*.

PROBLEM V.

How to Deſcribe an Inclin'd Dial, *Declining from the South.*

Plate 23. *Fig.* 62. If you Work upon the Upper Surface of an Inclin'd Declining Plain, (where you have, as uſual, determin'd the Foot of the Style AB, at the Point A, and found the Zenith C, the Vertical of the Plain LC, and the Horizontal EF, whoſe Dividing Center L is found as in *Probl.* 3.) draw at the Dividing Center L, the Angle DLG of the Declination of the Plain (on the Right, or on the Left, according as the Plain declines towards the Eaſt or towards the Weſt) to have upon the Horizontal EF the South Point G, thro' which, and thro' the Zenith C, you muſt draw the Meridian CG, upon which muſt be mark'd the Center of the Dial, thus:

Having drawn thro' the Foot of the Style A, the Indefinite Line AK, perpendicular to the Meridian CG, and the Line AO, equal to the Length of the Style AB, and parallel to the ſaid Meridian CG, ſet off the Hypotenuſe IO, upon the Perpendicular AK, from I to K, which Point will be the Dividing Center of the Meridian CG, from which conſequently you muſt draw to the Zenith C, the Line KC, to make with it downwards the Angle CKM of the Complement of the Height of the Pole, to have the Right-line KM, which here meets the Meridian below the Zenith at the Point M, which will be the Center of the Dial, from which you muſt draw the Hour-lines thro' the Hour-points, which muſt be mark'd upon the Horizontal EF, by means of an *Horizontal Dial*, whoſe Center muſt be applied at the Point L, the Dividing Center of the Horizontal-line, in ſuch manner, that its Meridian may fall in with the Line of Declination LG.

You may alſo mark the Hour-points upon the Equinoctial-line, which muſt be drawn as in the *Vertical Declining Dial*, which we will again Repeat here. Having drawn thro' M the Center of the Dial, and the Foot of the Style A, the Subſtylar MR, draw to it thro' the ſame Foot of the Style A, the Perpendicular AP, equal to the Length of the Style AB, and draw the Axis PM, to draw to it at the Point P, the Perpendicular PN, which will upon the Subſtylar give the Point N, which you muſt draw to it the Perpendicular SH, which will be the Equinoctial-line: Or elſe at the Point K, (the Dividing Center of the Meridian MG,) make with the Line KC the Angle CKQ, of the Height of the Pole above the Horizon, to have upon the Meridian the Point Q, thro' which, and thro' H the Six a Clock-point muſt be drawn the Equinoctial SH,

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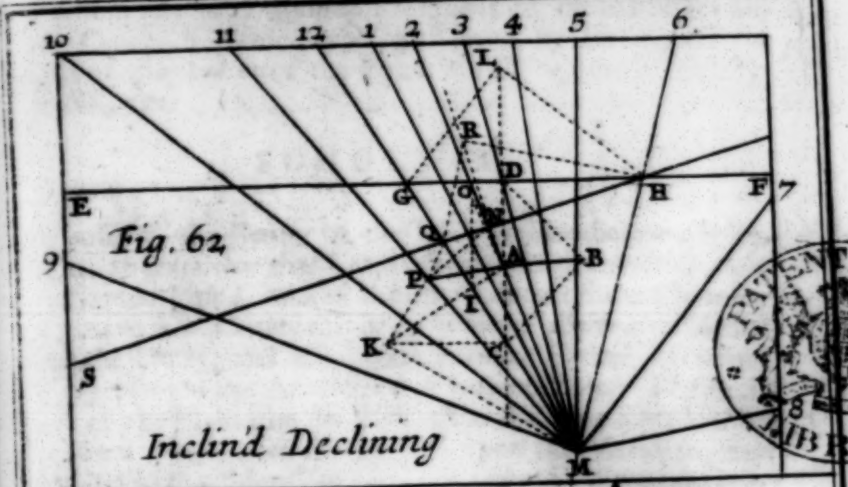
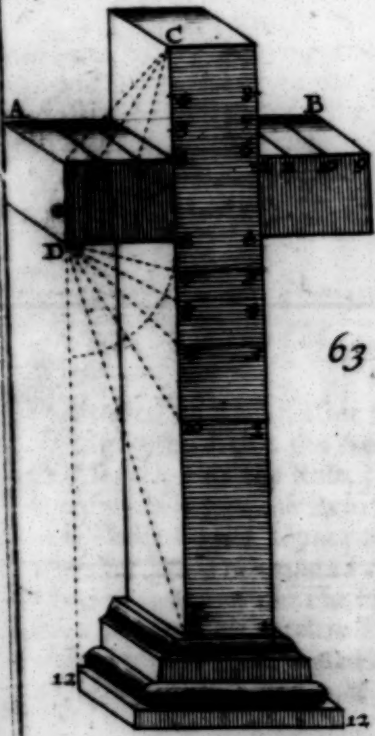


Fig. 62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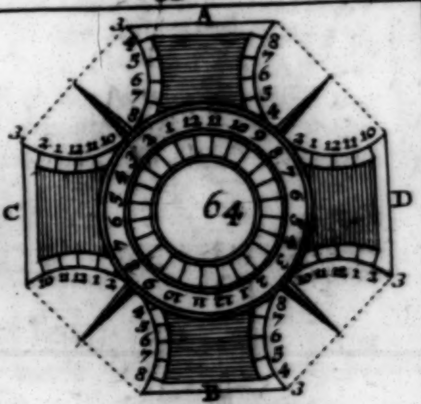
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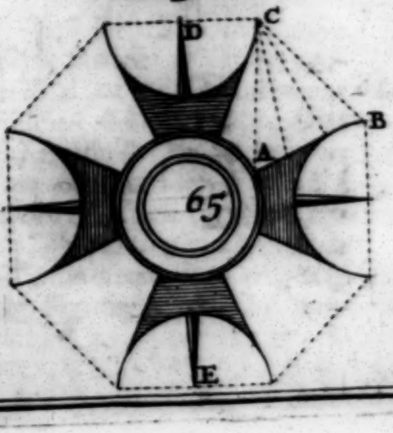
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H

1 MAY 70

SH, which you may divide into Hours by means of its Dividing Center K, which you will have by ſetting off the Length of the *Radius* of the Equator NP, upon the Subſtylar at NR, *&c.* *Plate* 23. *Fig.* 62.

SCHOLIUM.

Becauſe M, the Center of the Dial, is here below the Zenith C, it appears that that Center repreſents the Arctick Pole, or the Upper Pole: And if the Inclination of the Plain had been greater, the Center might have been above the Zenith C, and the Horizontal EF, and then that Center M wou'd have repreſented the Antarctick or Lower Pole. The Inclination of the Plain may be ſuch, that the Line KM, which determines the Center of the Dial upon the Meridian, may be parallel to that Meridian, and then the Dial will have no Center, its Plain being parallel to the Axis of the World, and it will conſequently be a Polar Declining, where the Hour-lines will be drawn parallel to the Meridian, thro' the Hour-points mark'd upon the Horizontal EF, or upon the Equinoctial SH, which in this caſe will go thro' the Foot of the Style A.

PROBLEM VI.

How to draw an Inclin'd Dial, *Declining from the North.*

THis Dial is drawn after the ſame manner as the foregoing, except that the Angle CKM, of the Complement of the Elevation of the Pole, (which in the foregoing Dial has been made below the Zenith C) muſt be made in this above it, becauſe the Upper Surface of the Plain (which I ſuppoſe that we work upon) faces the Arctick Pole, or the Pole which is above the Horizon. This is ſo plain by what has been ſaid in *Probl.* 2. that it wou'd be needleſs to ſay any more about it, or give a particular Figure, eſpecially ſince theſe Dials are very little in Uſe. *Fig.* 62.

PROBLEM VII.

How to draw a Dial upon a Croſs.

Plate 23. Fig. 63. IF CE, the Tree of the Croſs, be fix'd in a Poſition agreeable to the Height of the Equator above the Horizon, by inclining it towards the South, by the Complement of the Height of the Pole, in ſuch manner, that the Arm or Croſs-piece AB, may alſo be parallel to the Equator, and upon this Arm AB, a *Polar South Dial* be drawn, by applying to the End C, which will be as the End of the Style, a Circle of Paſt-board, divided into Parts of 15 Degrees, and upon the Tree CE be drawn a *South-Eaſt* and a *South-Weſt Dial*, making at the Point D, which will alſo be as the End of a Style, Angles of 15 Degrees, *&c.* you will, upon this Croſs thus fix'd, know the Hours when the Sun Shines, by the Shadow of the Tree of the Croſs upon the Arm AB, and by the Shadow of the Arm AB upon the Tree CE.

Fig. 64. The Croſs may be cut into an Octogon, as in *Fig.* 64. which is as ſoon Underſtood as Seen, where you ſee that each concave Semi-circle, whoſe Center may ſerve as the End of a Style, has been divided into 12 equal Parts, and the whole Circle, which is in the middle of the Croſs, into 24 equal Parts, to be an *Equinoctial Dial*; the one an Upper, and the other a Lower *Equinoctial Dial.* You may alſo draw upon each of the oppoſite and parallel Faces A, B, a *Polar South Dial*, One an Upper, and the other a Lower; and upon each of the other Two C, D, a *South Dial*, which is alſo Polar; the one looking Eaſt and the other Weſt.

Fig. 65. This Octogon may alſo be made as *Fig.* 65. repreſents, where you may alſo add the Hours upon the Sides, as upon the Side AB, making at the Point C, which will be inſtead of the End of a Style, Angles of 15 Degrees, beginning by the Line CA, which muſt be parallel to the Meridian of the *Equinoctial Dial*, or to the Line DE, which joins the Centers of the oppoſite Semi-circles.

PROBLEM VIII.

How to draw an Univerſal Equinoctial Dial.

Plate 24. Fig. 66. TO an *Horizontal Dial* made for any particular Latitude, (upon a Square, as ABCD, with a Magnetick-Needle in the middle) is uſually added an *Equinoctial Dial*, deſcrib'd upon

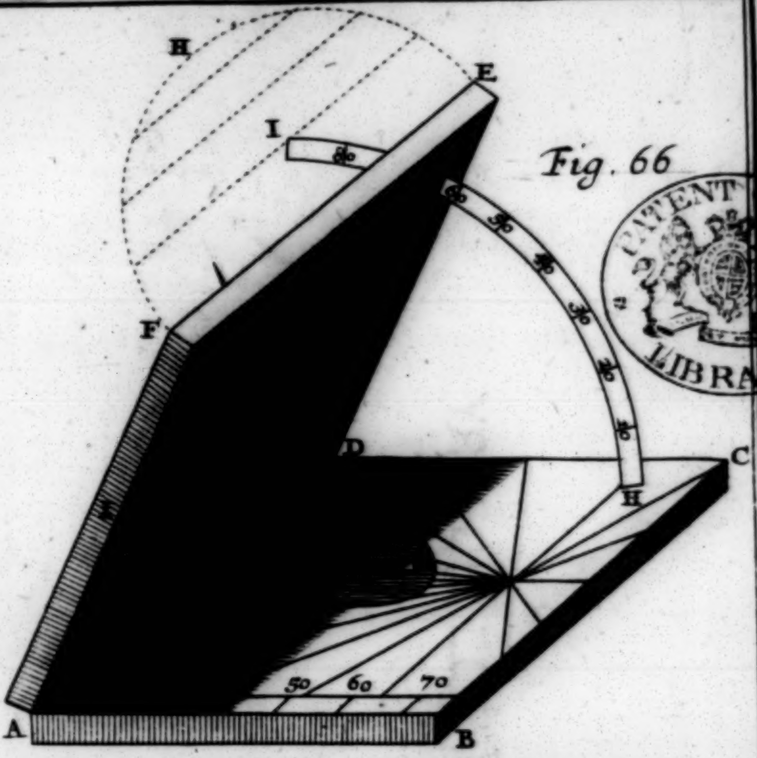
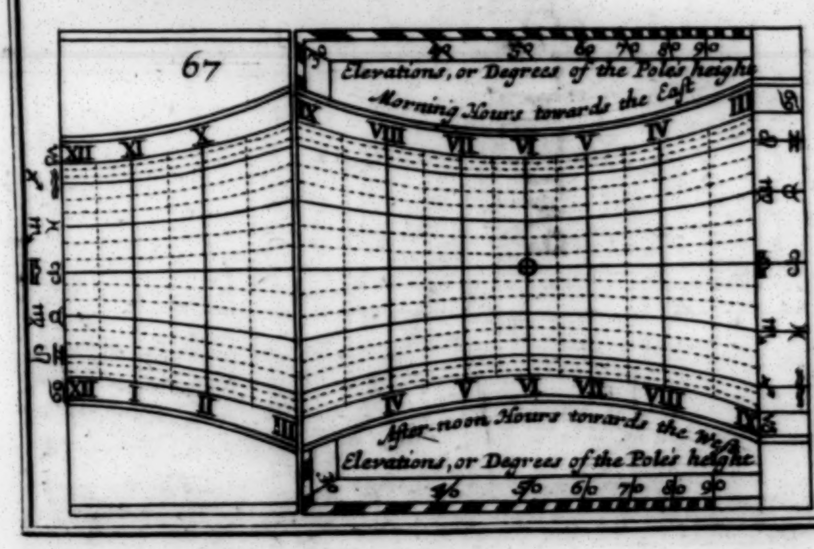


Fig. 66



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90
Elevations, or Degrees of the Poles height
Morning Hours towards the East
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XI
X
IX
VIII
VII
VI
V
IV
III
XII
I
II
III
IV
V
VI
VII
VIII
IX
After-noon Hours towards the West
Elevations, or Degrees of the Poles height
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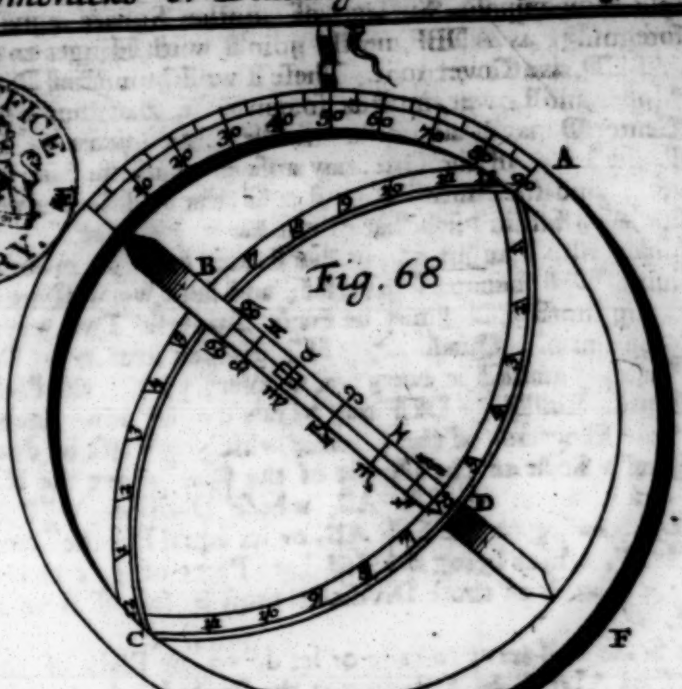

Fig. 68

A
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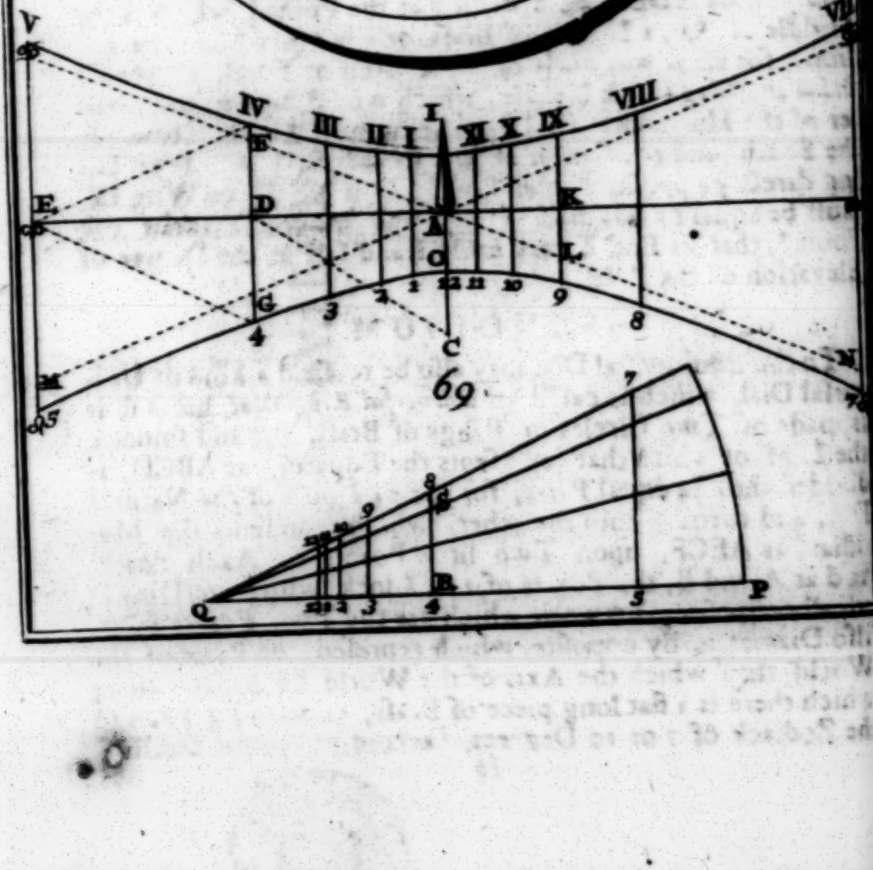

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upon the Two oppoſite Surfaces of another Square equal to the foregoing, as ADEF, neatly join'd with Hinges to the Firſt ABCD, as a Cover to it. Theſe Two Equinoctial Dials, both Upper and Lower, muſt be ſo contriv'd, that their common Center O may be about the middle of the Square ADEF, and all the Lines of the One may anſwer to all the Lines of the Other, and the Meridian muſt at G (the middle Point of the Line AB) fall in with that of the *Horizontal Dial*.

Plate 24. *Fig.* 66.

We have alſo Taught in another Place, how to make an Horizontal Dial become Univerſal; and here we will ſhew, that the Equinoctial Dial may be made Univerſal Two ways: Firſt by means of a Quadrant, as HI, divided into its 90 Degrees, or only mark'd at every 10, or every 15, that the Plain of the Equinoctial Dial ADEF may be rais'd or let down, according to the Elevation of the Equator, which may alſo be done by help of a Scale of the Height of the Pole above the Horizon, drawn upon the ſide AB, whoſe Diviſions will be found by drawing on the Side AB, or its equal EF, the Semi-Circle HEF, which being divided into Parts of 5 or 10 Degrees, the Chords of thoſe Diviſions muſt be ſet off from A towards B.

This Scale will ſerve to raiſe or let down the Plain of the Equinoctial ADEF, by faſtening at the Point I, which is the Middle of AF, a Needle of Braſs, or any other Metal except Iron; for if it was made of either Iron or Steel, it wou'd draw the Magnetick Needle, which wou'd hinder the Center of the Horizontal Dial from being turn'd exactly towards the South, and the Center of the Equinoctial Dial from being directly oppoſite to the Pole. This Needle or Wire IK, muſt be equal to AI, half of AF, and moveable about the Point I, that its End K may eaſily ſtand faſt at the Degree of Elevation of the Pole, *&c.*

SCHOLIUM.

To the Equinoctial Dial may alſo be referr'd a kind of Univerſal Dial, which is call'd an *Univerſal Ring-Dial*, becauſe it is made of Two Circles or Rings of Braſs, flat and ſmooth, the Leaſt of which that repreſents the Equator, as ABCD, is divided into 24 equal Parts, for the 24 Hours of the Natural Day, and turns within the other, which repreſents the Meridian, as AECF, upon Two little Points or Axels neatly fix'd at A and B, the Points of 12 a Clock, which are Diametrically oppoſite; between which are the Two Points E, F, alſo Diametrically oppoſite, which repreſent the Poles of the World, thro' which the Axis of the World EF paſſes, along which there is a flat long piece of Braſs, containing Parts of the Zodiack of 5 or 10 Degrees, (according to the Declina-

Plate 25. *Fig.* 68. tion of thoſe Points of the Zodiack, which is meaſur'd upon a Circle, whoſe Diameter is equal to the inner Diameter of one of the Two ſaid Circles) which is open along the Middle of the Axis, for a little piece of Braſs to ſlide up and down it, which Piece has a little Hole in the Middle of it, which muſt be ſet at the Degree of the Sign in which the Sun is when you wou'd know the Hour. The Quadrant AE muſt be divided into its 90 Degrees, that you may ſuſpend the Inſtrument from the Degree of the Latitude of the Place where you are, and then the Ray of the Sun which goes thro' the little Hole of the ſliding Piece, which is ſet at the Sun's place in the Zodiack, BD will ſhew upon the middle of the thickneſs of the Equator, the Hour requir'd, when that Equator is perpendicular to the Plain of the Meridian. As this Dial is very common, I ſhall ſay no more about it.

PROBLEM IX.

How to draw an Univerſal Polar Dial.

Plate 24. *Fig.* 67. THE Two Polar Dials which we have deſcrib'd upon a Croſs in *Probl.* 7. may eaſily be made Univerſal, if you incline the Croſs by means of a Thread and Plummet hanging from the Center of a graduated Circle, according to the Latitude of the Place where you are, ſo as to give it a Poſition parallel to the Equator: But a Polar Univerſal Dial may more conveniently be drawn upon Two Leaves of Paſt-board, or Ivory, or any other ſolid Matter neatly join'd together, ſo as they may open at Right-Angles, and ſhut again at pleaſure.

In one of theſe Two Plains is drawn a Polar South-Dial, and in the other a South-Dial, which has on each ſide the Scale of Latitude, or Degrees of the Height of the Pole, having its Center in the middle Point of the Line of 3 and 9 a Clock, where you ſee the Two Characters ♈ ♎, from whence a Thread muſt hang with its Plummet upon the Degree of the Pole, to give the Dial its due Elevation when you wou'd know the Hour.

Each of thoſe Dials ought, beſides the Equinoctial which goes thro' the Middle, to have the Arches of the other Signs, whoſe deſcription will be taught in the following Chapter. The Arches will direct you how to turn the Dial to the Right Point of the Compaſs, when you wou'd uſe it, by turning it (when the Thread cuts the Degree of the Height of the Pole) 'till the End of the Style, which may ſerve for both Dials when they are at Right-Angles to one another, caſts its Shadow upon the Degree of the Sign in which the Sun is.

When theſe Two *Dials* are whole; for here we have only half of the Polar South-Dial, (which is enough) the Two Plains muſt be join'd by their Ends by a Thread or Silken-ſtring faſten'd to the Middle Points between 3 and 9 in each Dial, in ſuch manner, that the length of the String may be equal to the Diagonal of a Square, whoſe Side is equal to the Length of the Dial; for thus the String will keep the Two Plains at Right Angles, and if in its Middle Point you have a Knot or ſmall Bead, that Knot or Bead will repreſent the End of a Style for each Dial, and mark the Hours by its Shadow.

Plate 24. *Fig*. 67.

The only inconvenience of this Dial is, that you muſt caſt your Eye upon Two Places at One Time, *viz*. the Degree of Latitude of the Place where you are, and the Degree of that Sign in which the Sun is.

CHAPTER V.

Of the Deſcription of the Arches of the Signs, *and other Circles of the* Sphere, *in* Dials.

BEſides the Hour-lines, which ſhews the Hours by the Sun's Rays, you may for Ornament add on the Dial, the Arches or Parallels of the Signs, to know the Sun's Place in the Zodiack; the Diurnal and Nocturnal Arches, to know the Length of Days and Nights, and the Hour of the Sun's riſing and ſetting, the *Italian* and *Babyloniſh* Hours, the Time from the riſing and ſetting of the Sun, and even the *Jewiſh* and *Antique* Hours, to know the Hours after the manner of the *Ancients*, who us'd to divide the Day into 12 equal Parts: And laſtly the *Azimuths*, to know the Sun's Diſtance from the Eaſt, and the *Almicanters*, to know at all times the Height of the Sun above the Horizon.

PROBLEM I.

How to draw the Arches of the Signs upon Polar Dials.

TO draw the Arches of the Signs upon a *Polar Dial*, you muſt find their Points upon each Hour-line, to join them by Curves, which will repreſent thoſe Parallels, where conſequently the End of the Shadow of the Style will fall, when the Sun is in thoſe Parallels.

Plate 25. *Fig.* 69.

For this end you muſt prepare the *Triangle of Signs*, or only half of it, as PQR, of which the Line QP muſt be the Equator, and the Line QR one of the Tropicks, which makes an Angle of 23 Degrees and 30 Minutes with the Equator QP, and you muſt find the Dividing Center of each Hour-line, to apply upon it the Center Q of the Triangle of Signs, and mark upon each Hour-line the Points of the Signs, thus:

Firſt to mark upon the Subſtylar AC the Points of the Signs, you muſt ſet off the Length of the Style AI upon the Equinoctial, from the Foot of the Style A to the Point K, which will be the Dividing Center of that Subſtylar, and where conſequently you muſt apply the Center Q of the Triangle of Signs, in ſuch manner, that the Equator QP may fall upon the Equinoctial-line, and then the *Radii* of the other Signs will cut the Subſtylar at the Points requir'd, whoſe Diſtances from the Equinoctial muſt be ſet off on either ſide upon the ſaid Subſtylar, to have upon it the Points of the other Signs.

Then to have the Points of the ſame Signs upon the other Hour-lines, *For Example*, upon the Line FG, which cuts the Equinoctial at the Point D, take upon the Subſtylar the Line AC, equal to the Length of the Style AI, to have at C the Dividing Center of the Equinoctial, and ſet off the Hypotenuſe CD, from D upon the ſaid Equinoctial to E, which will be the Dividing Center of the Hour-line FG, where you muſt therefore draw the Angles of the Declination of the Parallel which you wou'd deſcribe, *viz.* The Angles DEF, DEG, of 23 Degrees and 30 Minutes each, for each Tropick, which will be done, by applying as before, at the Point E, the Center Q of the Triangle of the Signs, *&c.*

SCHOLIUM.

This way of working is good when you deſcribe a Dial upon a great Plain, as when you draw a *South-Dial* upon a Wall, (where nothing is to be drawn above the Horizontal-line) which will ſhew the Hour of the Sun's Riſing and Setting in each Sign, when the Signs are drawn: But when you work upon a little Plain, as when you deſign a Dial upon Paper, inſtead of carrying or ſetting off the Triangle of Signs upon Paper, it is better to ſet off the Hour-lines upon the Triangle of Signs, thus:

To ſet off, *for Example*, the Hour-line FG, whoſe Equinoctial-point is D upon the Triangle of Signs, ſet off the Diſtance from the Equinoctial-point D, to the Dividing Center C, of the Equinoctial in the Triangle of Signs, from Q upon the Equator QP to R, and draw thro' the Point R to the

the ſaid Equator QP, the Perpendicular RS, which will be divided into Points by the *Radij* of the Signs, which you muſt ſet off, taking them from R, upon the Hour-line FG on each ſide from its Equinoctial-point D, *&c.*

Plate 25. *Fig.* 69.

This is the way of drawing upon a *Polar Dial* the Parallel of the Riſing and Setting of the Sun, if you know their Declination, which will be done by means of the Triangle of Diurnal and Nocturnal Arches. Knowing alſo the Sun's Declination for any Day of the Year, you may repreſent its Parallel upon the Plain. Thus you may upon a Dial ſet down the chief Unmoveable Feaſts of the Year; the Day of ſuch a King's Birth; of the taking of ſuch a Town, always obſerving whether the Declination of the Sun for that Day is Northern or Southern, to mark it accordingly in the Dial.

But the Arches of the Signs may be very eaſily and exactly drawn upon a *Polar South-Dial*, and in imitation of it upon a *Meridian Dial*, taking the Six a Clock-line for the Meridian, by means of the following Table, which ſhews the Diſtances of the Parallels of the Signs upon the Hour-lines and half Hour-lines, on either ſide from the Equinoctial, in Parts of the Style, ſuppos'd to be divided into a Thouſand Parts.

A Table of the Diſtances of the Arches of the Signs upon the Hour-lines of a Polar Dial, *from the Equinoctial, for a Style divided into One Thouſand Parts.*

	XII. *	I. *	II. *	III. *	IV. *	V. *	
♉ ♍	203.205	211.220	235.256	288.334	407. 532	786.1559	♏
♊ ♌	368.371	381.398	425.464	520.604	736. 961	1427.2819	♐ ♒
♋	435.439	450.470	502.548	615.705	870.1136	1678.3333	♑
	XII. *	XI. *	X. *	IX *	VIII. *	VII *	

This Table has been Calculated for the Meridian Line by this Analogy:

As the whole Sine:
To the Tangent of the Declination of the Parallel::
So the Length of the Style:
To the Diſtance of the ſame Parallel.

And for the other Hour-lines, by this,

As the Square of the Whole Sine:
To the Rectangle under the Secant of the Horary Diſtance, and the Tangent of the Declination::
So the Length of the Style:
To the Diſtance requir'd::

Plate 25. *Fig.* 69 The Arches of the Signs may be drawn upon ſuch a Plain very eaſily, without having the Hour-lines, or Triangle of Signs, in the following manner.

Having thro' the Foot of the Style A, drawn the Two Lines AM, AN, which with the Subſtylar AC make Angles, each equal to the Complement of the Declination of the Parallel which you wou'd repreſent; ſet off the Length of the Stye AI upon the Equinoctial at AK, to have at K the Dividing Center of the Subſtylar AC, and draw thro' that Point K, to the Subſtylar AC the Parallel KL, whoſe Length, which is terminated by the Line AN, muſt be ſet off upon the Subſtylar AC, from A to O, to deſcribe thro' that Point O, from the Center A, between the Aſymptotes AM, AN, an Hyperbola, which will be the Repreſentation of the Parallel requir'd.

Plate 26. *Fig.* 70. If in a *Polar South-Dial*, ſeveral Horizontal-lines be drawn for ſeveral Latitudes, or Elevations of the Pole, (making at K, the Dividing Center of the Meridian AC, Angles with the Equinoctial, equal to the Complement of the Height of the Pole above the Horizon which you wou'd repreſent, to have upon the Meridian AC the Point of that Horizon, thro' which you muſt draw the Horizontal-line parallel to the Equinoctial) you muſt upon thoſe Horizontal-lines know the Hours of the Riſing and Setting of the Sun, on the Horizons which they repreſent, at the beginning of each Sign.

Thus to know at *Paris*, (where the Complement of the Height of the Pole is of about 41 Degrees) at what time the Sun riſes and ſets the longeſt Day of Summer; that is, when the Sun is in the beginning of ♋, you muſt at K, the Dividing Center of the Meridian, make the Angle AKP of 41 Degrees, and thro' the Point P, where the Line KP cuts the Meridian, you muſt draw the Horizontal-line RS, parallel to the Horizontal, or perpendicular to the Meridian: And becauſe, thro' the Point R, where that Horizontal cuts the Tropick of ♋, the Line of 4 in the Afternoon does paſs, that ſhews, that when the Sun is in the Tropick of ♋, it riſes on the Horizon of *Paris* at 4, and conſequently ſets at 8 in the ſame Horizon; where you ſee alſo, that when it is in the beginning of ♉ and ♊, it riſes about 5, and ſets about 7.

Likewiſe to know at what time the Sun riſes and ſets at the beginning of ♋, on the Horizon of *Alexandria* in *Egypt*, where the Complement of the Latitude is of about 59 Degrees, you muſt at the Point K make the Angle AKQ, of 59 Degrees, with the Line KQ, which gives upon the Meridian the Point Q, thro' which Point you muſt draw to it the Perpendicular TU, which will repreſent the Horizon of *Alexandria*, and ſhew that when the Sun is in the beginning of ♋, it riſes on that Horizon about 5, and ſo on.

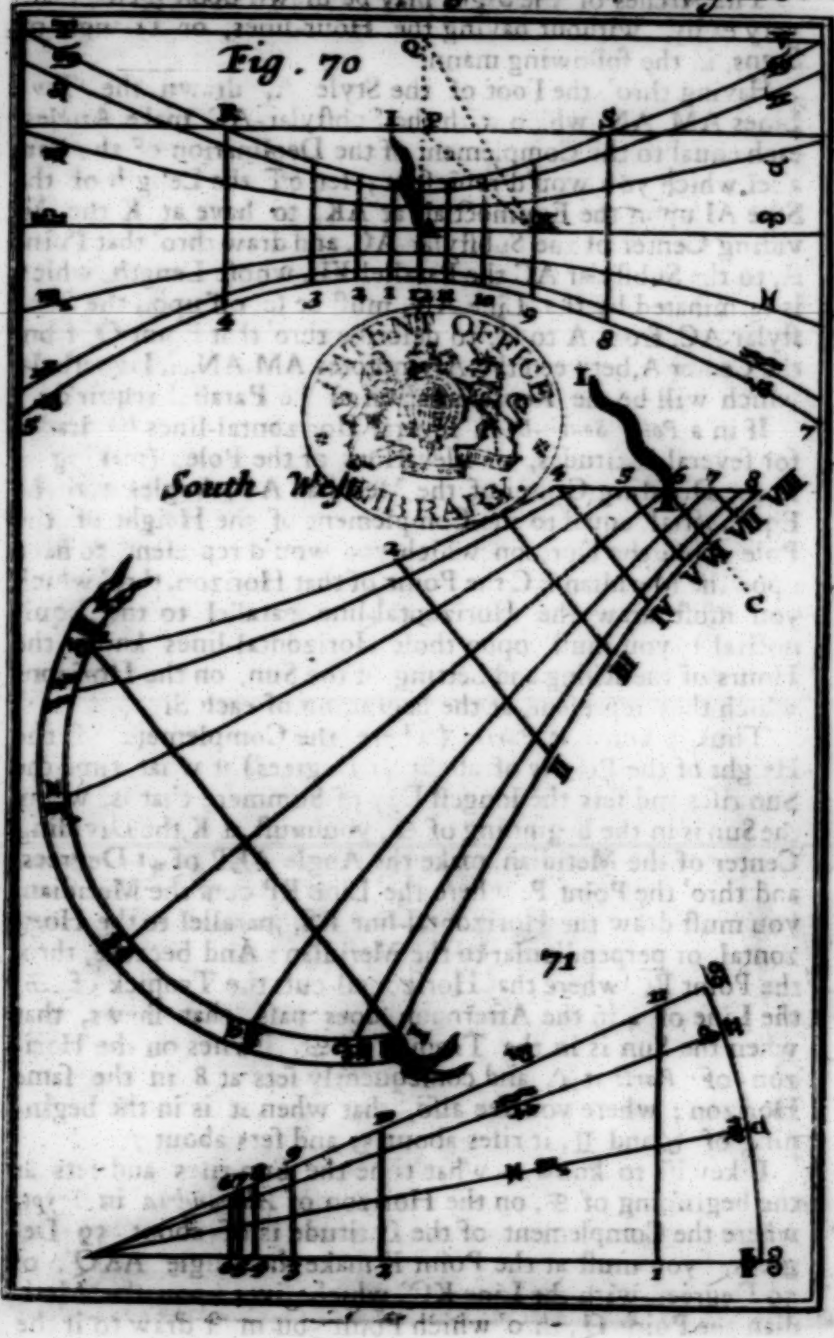
Fig. 70
Q
L
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South West
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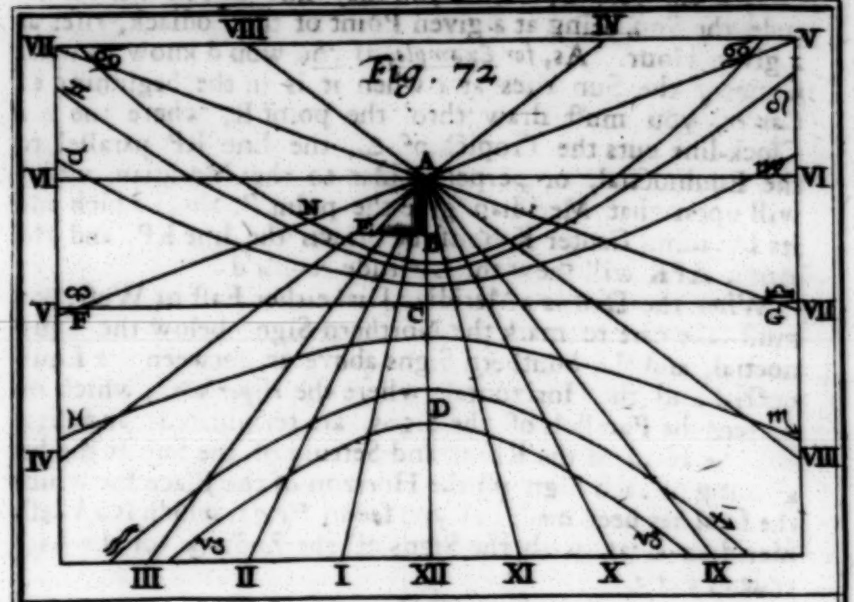
Gnomonicks or Dialling Plate 27 Pag. 119



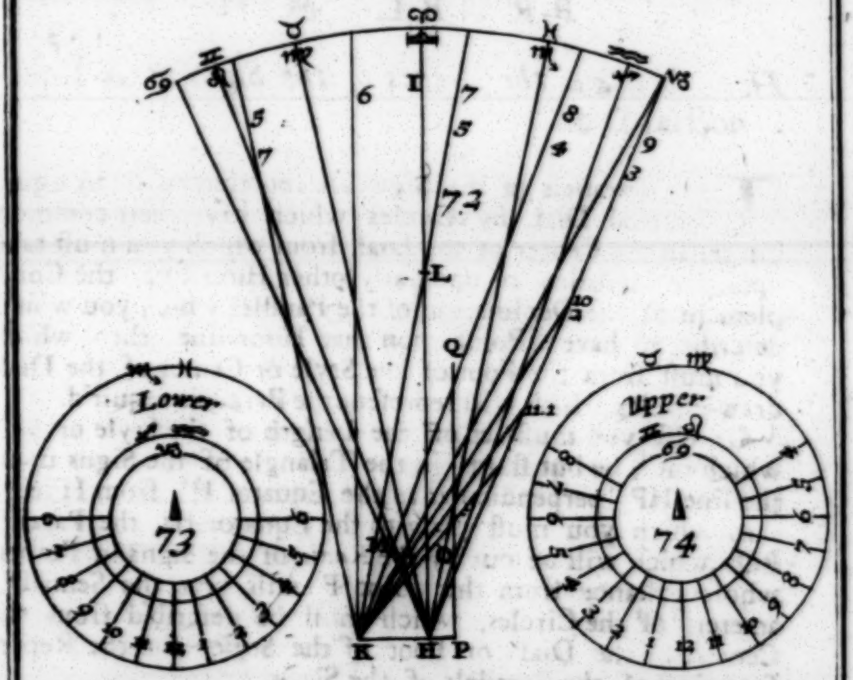


Plate 26. *Fig.* 70.

By a contrary Operation you may find out in what Latitude the Sun, being at a given Point of the Zodiack, *riſes* at a given Hour. As, *for Example*, if you wou'd know in what Country the Sun riſes at 4 when it is in the beginning of *Cancer*, you muſt draw thro' the point R, where the 4 a Clock-line cuts the Tropick of ♋, the line RP parallel to the Equinoctial, or perpendicular to the Meridian, which will upon that Meridian give the point P, thro' which and its Dividing Center K, muſt be drawn the line KP, and the Angle APK will ſhew the Latitude requir'd.

Fig. 71.

When the Dial is a Meridian Dial either Eaſt or Weſt, you muſt take care to mark the Northern Signs below the Equinoctial, and the Southern Signs above it, between the Equinoctial and the Horizontal, where the *Hyperbola's*, which repreſent the Parallels of the Signs, are terminated, and ſhew alſo the Hour of the Riſing and Setting of the Sun at the beginning of each Sign, on the Horizon of the Place for which the Dial has been made: as you ſee in *Fig.* 71. which is a Weſt-Meridian Dial, with the Signs of the Zodiack for the Latitude of *Paris*.

PROBLEM II.

How to draw the Arches of the Signs in an Equinoctial Dial.

Plate 27. *Fig.* 73, *and* 74.

THE Parallels of the Signs are repreſented in an Equinoctial Dial, by Circles which have their common Center in the Center of the Dial, from which you muſt take upon the Meridian, or upon any other Hour-line, the Complement of the Declination of the Parallel which you wou'd deſcribe, to have a Point upon that Hour-line, thro' which you muſt from the Foot of the Style or Center of the Dial, draw Circles which will repreſent the Parallels requir'd.

Or elſe you muſt ſet off the Length of the Style or *Axis* which muſt be but ſhort, in the Triangle of the Signs upon the line HP, perpendicular to the Equator HI, from H to P, thro' which you muſt draw to the Equator HI, the Parallel PQ, which will be cut by the *Radii* of the Signs in Points, whoſe diſtance from the point P will give the Semi-Diameters of the Circles, which muſt be deſcrib'd from the Center of the Dial, or Foot of the Style, for the Repreſentation of the Parallels of the Signs.

As this Dial has no Equinoctial Line, (its Plain being parallel to that of the Equator) it can contain but half of the Signs of the Zodiack; and if the Dial is drawn upon a Plain inclin'd to the Horizon of the Oblique Sphere directly

rectly towards the North, by the Degrees of the Complement of the Height of the Pole, that it may be parallel to the Equator, you muſt draw in that Hemiſphere the Northern Signs upon the Upper Surface of the Plain, and the Southern upon the Lower.

SCHOLIUM.

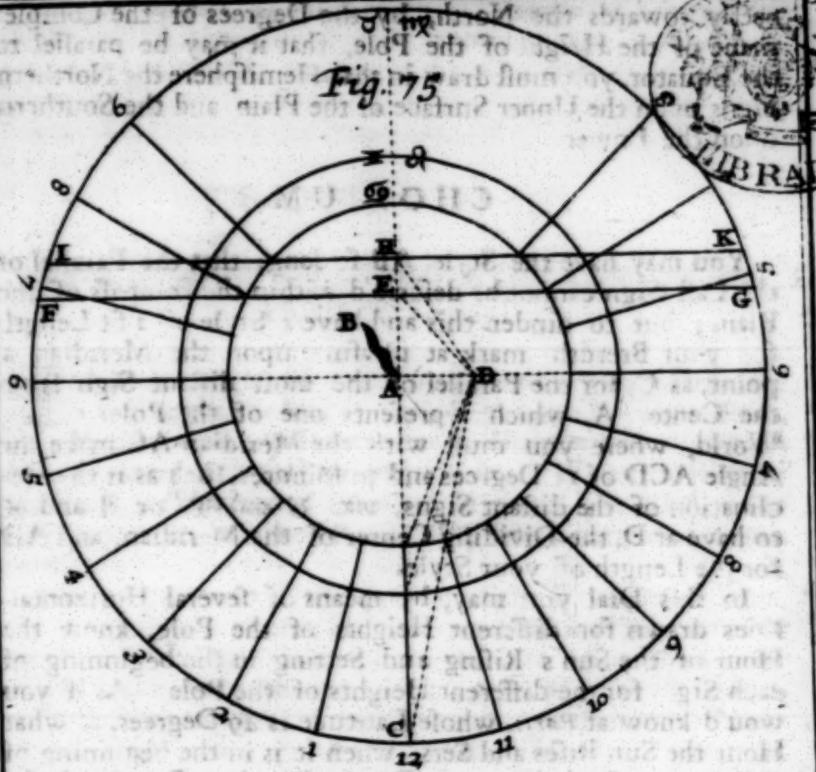
Plate 28. *Fig.* 75. You may have the Style AB ſo long, that the Parallel of the Laſt Sign cannot be deſcrib'd within the compaſs of the Plain; but to hinder this and have a Style of a fit Length for your Breadth, mark at pleaſure upon the Meridian a point, as C, for the Parallel of the moſt diſtant Sign from the Center A, which repreſents one of the Poles of the World, where you muſt with the Meridian AC make the Angle ACD of 11 Degrees and 30 Minutes, ſuch as is the Declination of the diſtant Signs, *viz.* ♉ and ♍, or ♏ and ♓ to have at D, the Dividing Center of the Meridian, and AD for the Length of your Style.

In this Dial you may, by means of ſeveral Horizontal-lines drawn for different Heights of the Pole, know the Hour of the Sun's Riſing and Setting in the beginning of each Sign, for the different Heights of the Pole. As if you wou'd know at *Paris*, whoſe Latitude is 49 Degrees, at what Hour the Sun Riſes and Sets, when it is in the beginning of ♋, you muſt at the point D, the Dividing Center of the the Meridian AC, draw the Angle ADE of 49 Degrees, and thro' the point E, where the line DE cuts the Meridian AC, you muſt to the Meridian AC draw the Perpendicular FG, which will repreſent the Horizon of *Paris*, and cutting the Tropick of ♋, at the Points of the 4th and 8th Hour, ſhews that the Sun on the Summer Solſtice riſes above the Horizon of *Paris* at 4, and Sets at 8 a Clock.

Likewiſe to know at what Hour the Sun, being in the Summer Tropick, Riſes and Sets at *Stockholm*, where the Pole is about 60 Degrees above the Horizon; at the ſaid point D you muſt draw the Angle ADH of 60 Degrees, with the line DH, which will upon the Meridian AC give the point H, thro' which you muſt draw to the ſaid Meridian AC, the Perpendicular IK, which will repreſent the Horizon of *Stockholm*, and which cutting the Tropick of ♋ at the points of 3 and 9, ſhews that the Sun, on the Day of the Summer Solſtice, Riſes at *Stockholm* at 3 and Sets at 9.

As we have made uſe of the Declination of the Signs, or Triangle of Signs, to repreſent the Parallels of thoſe Signs in this Dial: So likewiſe may you draw in it the Parallels of the Riſing and Setting of the Sun, or the Length of the

Fig. 75



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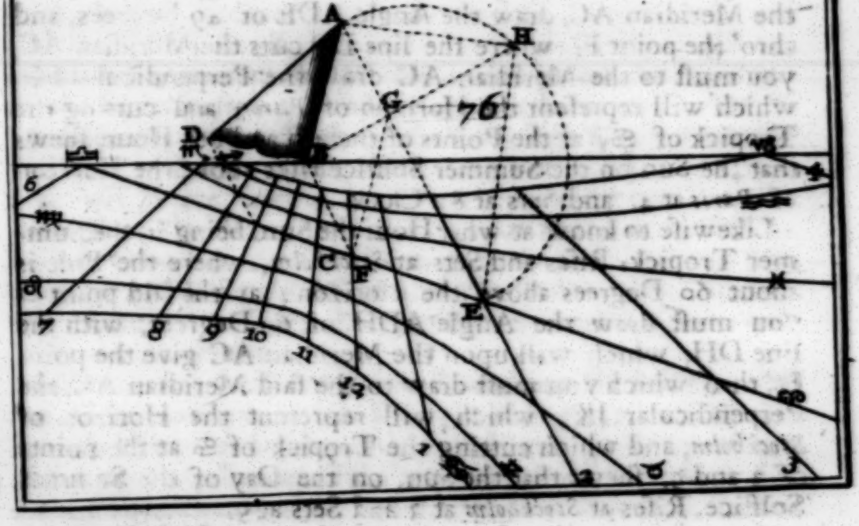
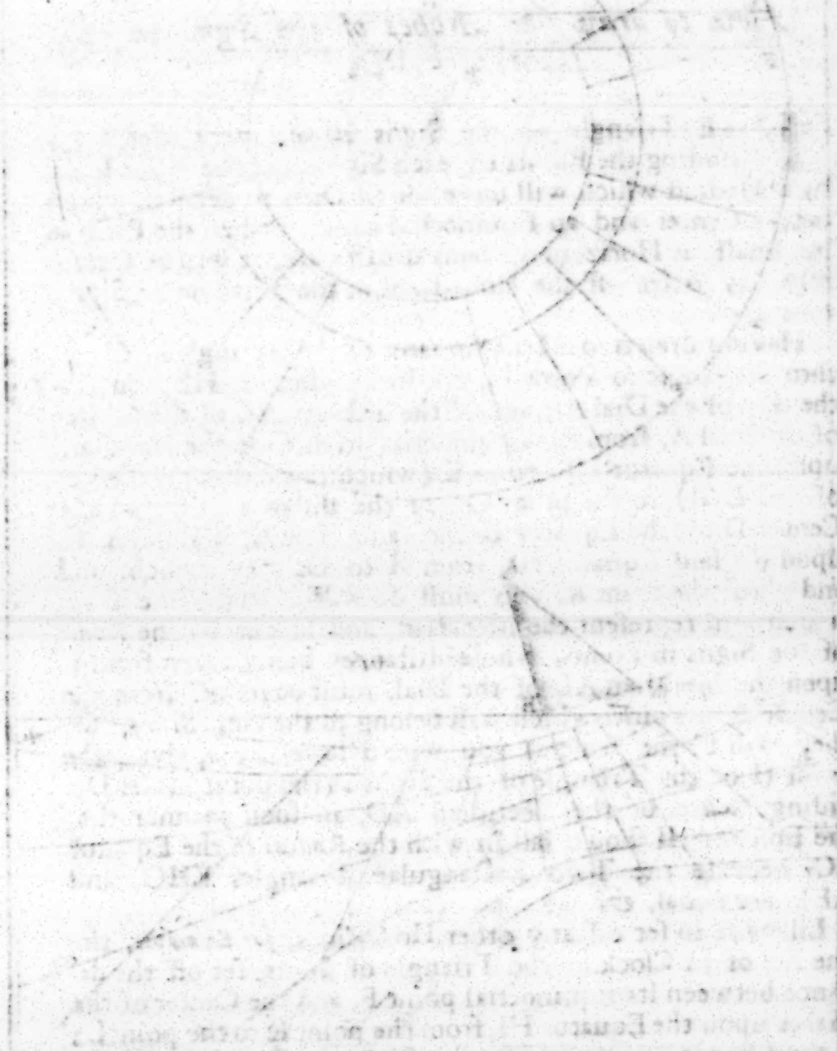


[illegible] Angles, [illegible] the Triangle [illegible] and N[illegible] without knowing them; you may more easily [illegible] of [illegible] the Center A [illegible] thro' the Point where the [illegible] Hour [illegible]



[illegible] Length [illegible] the Signs [illegible] [illegible] setting [illegible] each Sign [illegible] which will [illegible] and [illegible] [illegible] Hour [illegible]

[illegible] the [illegible] Dial [illegible] the [illegible] upon the Equator [illegible] which [illegible] the [illegible] Center D [illegible] upon [illegible] and [illegible] the Signs [illegible] upon [illegible] the Dial [illegible] [illegible] belong to [illegible] [illegible] the Equator [illegible] EC [illegible] Triangles EBC and AF [illegible]

[illegible] Clock [illegible] Triangle of [illegible] between [illegible] point F [illegible] the Center of the [illegible] upon the Equator [illegible] from the point [illegible] the point [illegible] between [illegible] and the Center of the Equator D, upon the Equator [illegible] from the point [illegible] to the point [illegible] which you [illegible] draw [illegible] the line [illegible] which will represent the Line of [illegible] Clock, and which will be cut by the Rays of the Signs [illegible] Points, whose distances

Days and Nights, by means of the Triangle of Diurnal and Nocturnal Arches; or without knowing their Declination, you may more eaſily repreſent thoſe Parallels by deſcribing from the Center A, Circles thro' the Point where the Horizontal AC is cut by the Hour-lines, *&c.* *Plate* 28. *Fig.* 75.

PROBLEM III.

How to draw the Arches of the Signs on an Horizontal-Dial.

THE Triangle of the Signs is alſo very uſeful for finding the Points of each Sign upon the Hour-lines, by a Method which will ſerve for all Dials in general, which have a Center and an Equinoctial Line. When the Plain is but ſmall, as Horizontal Plains uſually are, it will be a more eaſy way to ſet off the Hour-lines in the Triangle of Signs, thus:

Plate 27. *Fig.* 72. Having drawn to HI the Equator of the Triangle of Signs, thro' its Angle or Point H, the Perpendicular HK equal to the *Axis* of the Dial AE, ſet off the diſtance AC of the Center of the Dial A, from the Equinoctial point C of the Meridian, upon the Equator HI, from K (which repreſents the Center of the Dial) to the point O; or the diſtance CD from the Center D of the Equator to the ſame Equinoctial point C, upon the ſaid Equator HI, from H to O, thro' which, and and thro' the point K, you muſt draw the Right-line K 12, which will repreſent the Meridian, and be cut by the *Radii* of the Signs in Points, whoſe diſtances being taken from K upon the Meridian AD of the Dial, muſt be ſet off from the Center A in Points which will belong to the ſame Signs, for they will be the ſame as you wou'd have, by applying the point H of the Triangle of the Signs at the point E, the Dividing Center of the Meridian AD, in ſuch manner that the Equator HI ſhou'd fall in with the *Radius* of the Equator EC, becauſe the Two Rectangular Triangles KHO, and AEC are equal, *&c.*

Likewiſe to ſet off any other Hour-lines, *for Example*, the line AF of 5 a Clock in the Triangle of Signs; ſet off the diſtance between its Equinoctial point F, and the Center of the Dial A upon the Equator HI, from the point K to the point L; or the diſtance between the Equinoctial point F, and the Center of the Equator D, upon the Equator HI, from the point H to the point L, thro' which you muſt draw from the point K the line KL, which will repreſent the Line of 5 a Clock, and which will be cut by the *Radii* of the Signs in Points, whoſe

diſtances

Plate 27. *Fig.* 72. diſtances from the point K muſt be ſet off upon AF the 5 Clock-line of the Dial, from its Center A, to have upon it the Points of the Signs, *for Example*, the diſtance KM from the Center of the Dial A to the point N, which will belong to the Tropick of ♋: And ſo on.

The Six a Clock-line, which has no Equinoctial Point, that is, which does not cut the Equinoctial FG, is drawn in the Triangle of Signs thro' the point K, parallel to the Equator HI; and the other Hour-lines which are after Six at Night, or before Six in the Morning, are repreſented in the Triangle of the Signs by Right-lines, which make with the Six a Clock-line, that is already drawn, Angles, equal to thoſe which the Hour-lines that are already drawn, make with the ſame Six a Clock-line.

SCHOLIUM.

We do not here give the Demonſtration of this Operation, becauſe it is eaſy to any One who has well underſtood the General Rule for marking the Points of the Signs upon an Hour-line, by means of its Dividing Center, which we will explain in the following Problem, where you will know that the Dividing Centers of all the Hour-lines are diſtant from the Center of the Dial by the whole Length of the *Axis*.

Neither will I demonſtrate, that the 8 a Clock-line of the Triangle of the Signs, is for the Horizon of *Paris*, and for any other where the Height of the Pole is 49 Degrees, parallel to the *Radius* of the ♑, becauſe when the Sun is at the beginning of ♑ it riſes at 8: And likewiſe that the Line of 9 a Clock in the Triangle of Signs is for the Horizon of *Stockholm*, and for any other where the Pole is 60 Degrees high, parallel to the ſame *Radius* of ♑, becauſe the Sun being in the Tropick of ♑, Riſes on that Horizon at 9 a Clock.

Whence it is eaſy to conclude, that the Hour-lines of the Triangle of Signs make, with the Six a Clock-line, Angles equal to the Declination of the Diurnal and Nocturnal Arches, or of the Parallels of the Riſing and Setting of the Sun at thoſe Hours, which will give us an eaſy way to ſet off the Hour-lines in the Triangle of Signs, without having the Equinoctial Line on the Dial. This is ſo eaſy, that we need not ſay any more about it.

PROBLEM IV.

How to draw the Arches of the Signs in an Erect Dial.

Plate 28. Fig. 76.

IF the Erect Dial has a Center, and an Equinoctial, you may draw on it the Parallels of the Signs as in the foregoing Dial: But as thoſe kind of Dials are uſually drawn upon great Plains, as upon Walls, and that it is not eaſy to ſet off the Hour-lines in the Triangle of Signs; you muſt make uſe of the Dividing Center of each Hour-line, to mark upon it the points of the Signs, or of the Diurnal and Nocturnal Arches, by means of their Declination, after this manner.

Firſt, to mark the points of the Signs upon the Subſtylar AC, whoſe Equinoctial-point is C, and Dividing Center is the end D of the *Axis* AD, apply at that end D, the Point or Angle of the Triangle of the Signs, in ſuch manner that its Equator may fall in with the *Radius* of the Equator CD, and then the *Radii* of the other Signs being produc'd with a Thread or Ruler, will upon that Subſtylar AC, give the points requir'd.

To mark the points of the Signs upon any other Hour-line, as, *for Example*, upon the 2 a Clock-line AE, whoſe Equinoctial-point is E, draw thro' the Foot of the Style B, to that Hour-line AE, the Indefinite Perpendicular BH, and the Parallel BF equal to the Style BD, and ſet off the Length of the Hypotenuſe FG, from G to the Point H, which will be the Dividing Center of the Line AE, and where conſequently you muſt apply the Angle, or Point of the Triangle of Signs, in ſuch manner, that its Equator may anſwer to the Equinoctial-point E, to have, as before, the points of the Signs upon the Hour-line AE, where it is cut by the *Radii* of the Signs, which are to be produc'd upon occaſion.

SCHOLIUM.

Becauſe the Arch AE is of 90 Degrees in Repreſentation, the Angle AHE, which meaſures it, muſt be a Right, and it will be eaſy to conclude, *by Lem.* 5. *Chap.* 1. that the Line AH is equal to the *Axis* of the Dial AD, which ſhews that all the Dividing Centers of the Hour-lines are diſtant from the Center of the Dial AC by the Length of the *Axis* AD, whence is drawn a more eaſy Method than the former, to find the Dividing Center of an Hour-line, *for Example*, of the line AE, whoſe Equinoctial-point is E, by deſcribing on the

part

Plate 28. *Fig.* 76. part AE, which is terminated by the Equinoctial and the Center of the Dial, the Semi-circle AHE, to ſet off in it the line AH equal to the *Axis* AD, and the point A will be the Dividing Center requir'd.

We ſhall not here ſpeak of the *Erect Meridian Dial*, which has no Center, becauſe ſince it is a Polar Dial, the Signs will drawn, as we have taught in *Probl.* 1. Neither ſhall we ſpeak of *Inclin'd Dials*, becauſe the Method of drawing the Signs upon 'em is not different from the foregoing. You will by practice find ſeveral ſhort Methods, which will be obvious to any One that thoroughly underſtands the Properties of the Dividing Center of a Line.

We ſhall only ſay, that when the Subſtylar falls in exactly with an Hour-line, as here it happens to be the ſame as the 10 a Clock-line, that 10 a Clock-line is to be look'd upon as the Meridian of an Horizontal Dial, after which you may draw the Dial, and add the Arches of the Signs as if it was an Horizontal Dial. But the way to know when the Subſtylar is exactly in an Hour-line, is to know whether the Declination of the Plain be juſt equal to ſome Horary-arch, as here, where it happens to be juſt 23 Degrees and 30 Minutes, ſuch as is the Horary-arch of 2 and 10 for the Latitude of 49 Degrees.

PROBLEM V.

How to draw the Babylonian *and* Italian *Hours in a Dial.*

THE Hours which we have hitherto deſcrib'd in Dials are call'd *Aſtronomical Hours*, becauſe Aſtronomers have always made uſe of 'em, and alſo *French Hours*, becauſe the *French*, and all the other Nations of *Europe*, make uſe of 'em at preſent. But *Babylonian Hours* are thoſe which are reckon'd from Sun-riſing one Day, to Sun-riſing the next, and *Italian Hours* are reckon'd from Sun-ſet to Sun-ſet, and are 24 in Number. All theſe Hours are call'd *Equal*, to diſtinguiſh 'em from the Old Hours, which were in Uſe among the *Jews*; whence they have been call'd *Antique* and *Jewiſh Hours*, which we alſo call *Unequal Hours*; becauſe ſince they are the 12th part of the *Artificial Day*, (which in the Oblique Sphere is different at different times) they muſt needs be Unequal, and certainly longer in Summer than in Winter.

Ha-

4ª

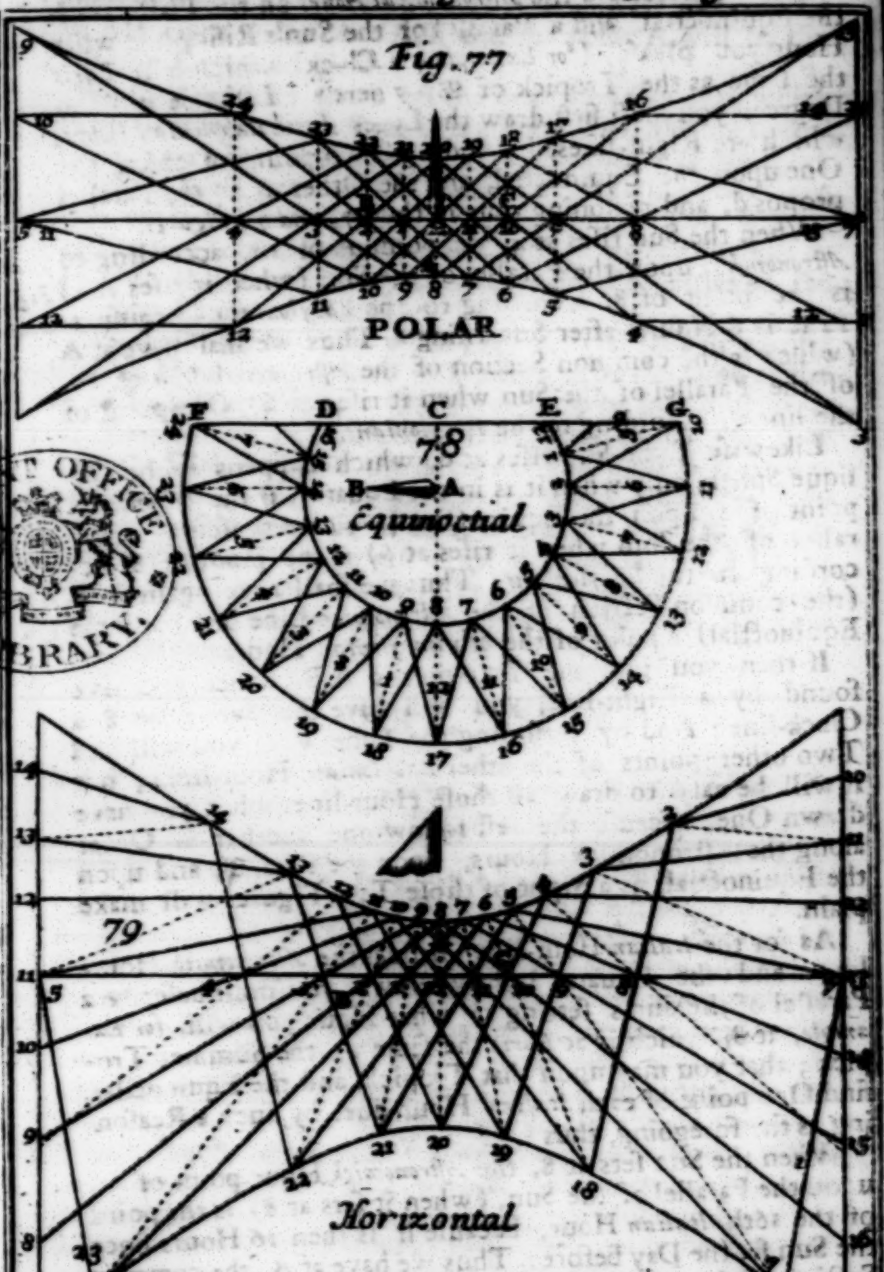


Fig. 77
POLAR
78
F
D
C
E
G
B
A
Equinoctial
79
Horizontal

Having deſcrib'd the *Aſtronomical Hours* on the Plain, with the Equinoctial, and a Parallel of the Sun's Riſing for what Hour you pleaſe, (*for Example* 4 a Clock) which is at *Paris* the ſame as the Tropick of ♋, where the Latitude is of 49 Degrees, you muſt firſt draw the Lines of the *Babylonian Hours*, which are Right-lines, by finding Two points of them, the One upon the Equinoctial, and the Other upon the Parallel propos'd, and reaſoning thus in the 77, *and* 79 *Figures*.

Plate 29. *Fig.* 77, *and* 79.

When the Sun riſes at 4, the point A of 12, according to *Aſtronomers*, upon the Parallel of the Sun (when it riſes at 4) is the point of 8, according to the *Babylonians*, becauſe at 12 it is 8 Hours after Sun-riſing: Thus we ſhall have at A (which is the common Section of the *Aſtronomick Line* of 12, of the Parallel of the Sun when it riſes at 4) One point of the line 8, according to the *Babylonians*.

Likewiſe if the Sun riſes at 6, which happens in the Oblique Sphere only when it is in the Equator, B the *Aſtronomick* point of 2 upon the Equinoctial (which repreſents the Parallel of the Sun when it riſes at 6) is the point of 8, according to the *Babylonians*: Thus we ſhall have again at B (the common Section of the *Aſtronomick* Line of 2, and the Equinoctial) a point of the 8th *Babylonian* Hour.

If then you join the Two points A, B, which you have found, by a Right-line, you will have the *Babylonian* 8 a Clock-line: And by reaſoning the ſame way, you will find Two other points of the other *Babylonian* Hour-lines; but it will be eaſy to draw all thoſe Hour-lines when you have drawn One, becauſe the reſt follow one another in Order along the Aſtronomick Hours, upon the Parallel, and upon the Equinoctial, as a Sight of theſe Two Figures will make plain.

As for the *Italian* Hour-lines, beſides the *Aſtronomick* Hour-lines, and the Equator lightly drawn, you muſt deſcribe a Parallel of the Sun's ſetting at what Hour you will, *for Example*, at 8, which is at *Paris* the ſame as the Summer Tropick; that you may upon that Tropick, and the Equinoctial, find One point of each *Italian* Hour-line, by ſuch a Reaſoning as the foregoing, thus:

When the Sun ſets at 8, the *Aſtronomick* Hour-point of 12, upon the Parallel of the Sun, (when it ſets at 8) is the point of the 16th *Italian* Hour, becauſe it is then 16 Hours ſince the Sun ſet the Day before: Thus we have at A (the common Section of the *Aſtronomick* 12 a Clock-line, and the Parallel of the Sun's ſetting at 8) a point of the *Italian* Hour-line of 16.

Like-

Plate 29. *Fig.* 77, *and* 79. Likewiſe when the Sun ſets at 6, which happens in the Oblique Sphere only at the Equinoctial times, the point C of the 10th *Aſtronomick* Hour upon the Equinoctial, which repreſents the Sun's Parallel (when it ſets at 6) is the point of the 16th *Italian* Hour: Thus we have again at C (the common Section of the *Aſtronomick* 10 a Clock-line, and the Equator) a point of the *Italian* Hour-line of 16, *&c.*

SCHOLIUM.

Becauſe in the Parallel Sphere the Sun does not riſe or ſet in the ſpace of 24 Hours, which happens alſo all over each cold Zone, one can reckon in it no other than *Aſtronomick* Hours. Nevertheleſs if in the Oblique, or in the Right Sphere, be deſcrib'd an Equinoctial Dial; the *Babyloniſh* and *Italian* Hours may be drawn by the ſame Principle as before.

Fig. 78. But as the Equinoctial Dial has no Equinoctial Line, you muſt in its room take the Sun's Parallel, when it riſes at 5. It is not neceſſary to write any thing beyond the Tropick, or the Horizontal-line FG, which determines upon the Meridian the Arch AC of the Height of the Pole in Repreſentation, and which cutting the *Aſtronomick* Hour-lines of 5 and 4, has given us the Two points E, G, thro' which we have drawn from the Pole, or Center of the Dial A, the Parallels of the riſing of the Sun at 4, and at 5.

The Lines of the *Babylonian* and *Italian* Hours give their Interſections the points of the *Aſtronomick* Half-hour, and cut the *Aſtronomick* Hour-lines in Points, thro' which paſs the other Diurnal and Nocturnal Arches; from whence may be deduc'd ſeveral ſhort Methods for the Deſcription of theſe Lines.

As the *Antique*, or *Jewiſh* Hours are of no great Uſe, and ſince it is not eaſy to repreſent them, becauſe ſuch Hour-lines are not quite Right-lines, it wou'd be needleſs to give a long account of 'em; we ſhall only ſay, That as they are almoſt Right-lines, you may draw 'em for every *Aſtronomick* Hour upon the Equinoctial-line, and for every Two Hours upon the Parallel of the Diurnal Arch of 24 Hours, beginning from the *Aſtronomick* Meridian-line, which will be the *Antique* 6 a Clock-line.

[illegible] which [illegible] in the [illegible] The [illegible] Fig. [illegible] the [illegible] celestial time [illegible] the [illegible] of the [illegible] first [illegible] the Equinoctial [illegible] (when [illegible] Thus we have [illegible]

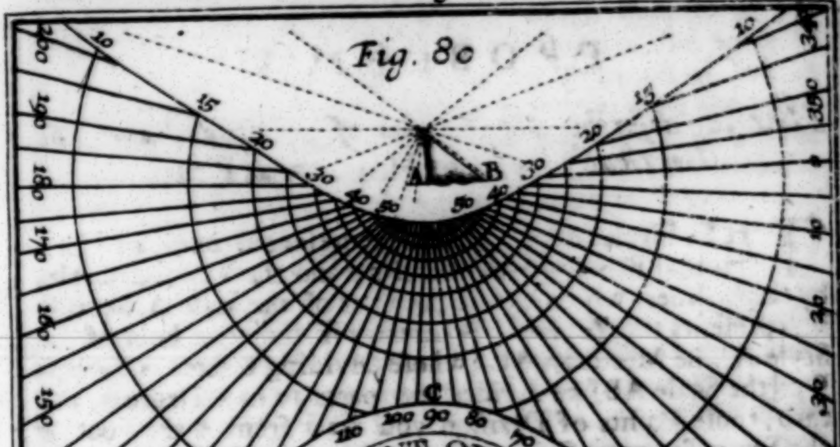
[illegible] VI.

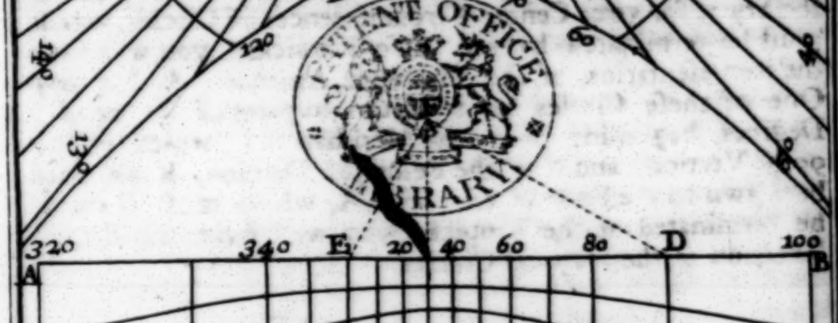
[illegible] Hours, which [illegible] in the [illegible] the same [illegible]

[illegible] Parallel [illegible] the Height of the [illegible]

[illegible] of [illegible] from [illegible] Dial, and [illegible] Hours [illegible] Lines [illegible] of Hour [illegible] we shall [illegible] may draw [illegible] Hour Lines [illegible] upon [illegible] line [illegible] Hours begins [illegible] Meridian [illegible] CHAP.

Fig. 80


A
B
C


E
D
A
B

81.

PROBLEM VI.

How to deſcribe the Circles of Altitude, and the Verticals, upon an Horizontal Plain.

Plate 30. *Fig*. 80.

THE Circles of Altitude are repreſented upon an Horizontal Plain by real Circles; and the Vertical Circles by Right-lines, which end at the Foot of the Style A, becauſe it repreſents the Zenith. Wherefore if from the Foot of the Style A, the Meridian AC (whoſe Dividing Center is the end B of the Style AB) be divided into parts 10 or 5 Degrees, and thro' thoſe Points of Diviſion you draw from A the Foot of the Style, for your Center, Circumferences of Circles, which muſt be terminated by the Two Tropicks; you will have the Repreſentation of the Circles of Altitude: And if any One of theſe Circles be divided into parts of 10, or of 5 Degrees, beginning from the Meridian AC, which is the 90th Vertical, and thro' the Points of Diviſion, Right-lines be drawn to the Foot of the Style A, which muſt likewiſe be terminated at the Tropicks, you will have the Repreſentation of the Vertical Circles.

PROBLEM VII.

How to deſcribe the Circles of Altitude, and the Verticals upon a Vertical *or* Erect Plain.

Fig. 81.

THE Circles of Altitude are repreſented upon a Vertical Plain, by *Hyperbola's*, as the Parallels of the Sun are upon a Plain parallel to the Equator: And the Vertical Circles by Right-lines perpendicular to the Horizontal, as the Hour Circles are upon a Plain parallel to the Equator. Wherefore if you divide the Horizontal AB into parts of 5 or 10 Degrees, (beginning from D the South Point, or from E the Six a Clock-point, thro' which the Prime Vertical goes, by means of a Circle divided into Degrees, whoſe Center muſt be apply'd at F the Dividing Center of the Horizontal-line AB) and thro' the Point of Diviſion as many Perpendiculars be drawn to the Horizontal, you will have the Repreſentation of the Vertical Circles.

For the Repreſentation of the Circles of Altitude, you muſt divide ſeparately into parts of 5 *or* 10 Degrees each Vertical, beginning from the Horizontal-line AB, (upon which

Plate 30. *Fig.* 81. which are all the Dividing Centers of the Lines which repreſent the Vertical Circles) and joyning all the Points which belong to the ſame Height by Curves, which will be *Hyperbola's*, and will repreſent the Circles of Altitude.

SCHOLIUM.

When the Plain does not decline, you muſt begin to divide the Horizontal from the Foot of the Style, (which in that caſe will repreſent the Point of 12, if the Dial be a South, or a North-Dial: Or the Point of Six if the Dial be an Eaſt or a Weſt-Meridian:) And ſet off the Diviſions of that Horizontal-line upon that Vertical which goes thro' the Foot of the Style, to have upon it the Points of the Circles of Altitude, thro' which you muſt draw *Hyperbola's* within their Aſymptotes, as has been taught in *Probl.* 1. for the Arches of the Signs.

You may alſo calculate a Table for a Style divided into 1000 equal Parts, as has been taught in *Probl.* 1. for the Parallels of the Signs; that you may take upon the Perpendiculars which repreſent the Vertical Circles, ſuch a Number of Parts as the Table will ſhew for each Circle of Altitude, and you will have the Points of thoſe Circles with all the exactneſs imaginable: you may alſo by Trigonometry calculate the Quantity of the other Lines and Angles, and by that means draw a Dial very exactly.

By means of ſuch a Calculation I formerly drew a Vertical Dial for 49 Degrees Latitude upon the Surface of a Wall, which declin'd from the South towards the Eaſt 60 Degrees, with the Two Tropicks, having exactly calculated the Quantity of the Angles and Lines, which was as follows:

Length of the Style	1000
Line of Declination	2000
Diſtance from the Center of the Dial to the Horizontal	2301
Part of the Meridian between the Horizontal and the Equinoctial	179
Part of the Horizontal between the Foot of the Style and the Six a Clock-point	577
Length of the Axis	3048
Radius of the Equator	1059
Part of the Subſtylar between the Center of the Dial and the Horizontal	2880
Part of the Subſtylar between the Horizontal and Equinoctial	347
Part of the Equinoctial between the Subſtylar and Horizontal	454

Angle

Angle of the Axis with the Subſtylar	19. 9′
Angle of the Subſtylar with the Meridian	36. 58′
Difference of the Meridians	69. 27′

Angles of the Hour-lines, *with the* Meridian.

I.	II.	XI.	X.	IX.	VIII.	*Subſt.*	VII.	VI.	V.	IV.
D.M.	D.M.	D.M.	D.M	D.M.	D.M.	D.M	D.M.	D.M.	D.M	D.M.
28.25	72. 3	14.36	23.31	29 37	34.50	36.58	40. 8	45. 6	51.37	60.55

Parts of the Horizontal-line, *comprehended between the* Meridian *and the* Hour-lines.

I.	II.	XI.	X.	IX.	VIII	*Subſt.*	VII.	VI.	V.	IV.
1245	7113	599	993	1309	1602	1732	1916	2309	2905	4136

Parts of the Hour-lines, *comprehended between the Center of the* Dial, *and the* Equinoctial.

I.	XII.	XI.	X.	IX.	VIII	*Subſt.*	VII.	VI.	V.	IV
7746	4039	3491	3321	3255	3230	3228	3232	3260	3336	3532

Parts of the Equinoctial, *between the* Meridian *and the* Hour-lines.

I	XI	X	IX	VIII	*Subſt.*	VII	VI	V	IV
4613	1100	1646	2012	2301	2428	2587	2882	3272	3861

Angles of the Hour-lines, *with the* Equator *of the* Triangle *of* Signs.

I	XII	XI	X	IX	VIII	*Subſt.*	VII	VI	V	IV
D.M.	D.M.	D.M.	D.M.	D.M.	D M.	D.M.	D.M.	D.M.	D.M.	D M
23.10	49. 0	60.50	66.37	69.30	70.41	70.48	70.36	69 15	66. 2	59 40

which are all the Dividing Centers of the Lines which represent the Vertical Circles) and joining all the Points which belong to the ſame Height by Curves, which will be *Hyperbola's*, and will repreſent the Circles of Altitude.

Plate 30.
Fig. 81.

SCHOLIUM.

When the Plain does not decline, you muſt begin to divide the Horizontal from the Foot of the Style, (which in that caſe will repreſent the Point of 12, if the Dial be a South, or a North-Dial: Or the Point of Six if the Dial be an Eaſt or a Weſt-Meridian:) And ſet off the Diviſions of that Horizontal-line upon that Vertical which goes thro' the Foot of the Style, to have upon it the Points of the Circles of Altitude, thro' which you muſt draw *Hyperbola's* within their Aſymptotes, as has been taught in *Probl.* 1. for the Arches of the Signs.

You may alſo calculate a Table for a Style divided into 1000 equal Parts, as has been taught in *Probl.* 1. for the Parallels of the Signs; that you may take upon the Perpendiculars which repreſent the Vertical Circles, ſuch a Number of Parts as the Table will ſhew for each Circle of Altitude, and you will have the Points of thoſe Circles with all the exactneſs imaginable: you may alſo by Trigonometry calculate the Quantity of the other Lines and Angles, and by that means draw a Dial very exactly.

By means of ſuch a Calculation I formerly drew a Vertical Dial for 49 Degrees Latitude upon the Surface of a Wall, which declin'd from the South towards the Eaſt 60 Degrees, with the Two Tropicks, having exactly calculated the Quantity of the Angles and Lines, which was as follows:

Length of the Style	1000
Line of Declination	2000
Diſtance from the Center of the Dial to the Horizontal	2301
Part of the Meridian between the Horizontal and the Equinoctial	179
Part of the Horizontal between the Foot of the Style and the Six a Clock-point	577
Length of the Axis	3048
Radius of the Equator	1059
Part of the Subſtylar between the Center of the Dial and the Horizontal	2880
Part of the Subſtylar between the Horizontal and Equinoctial	347
Part of the Equinoctial between the Subſtylar and Horizontal	454

Angle

Angle of the Axis with the Subſtylar 19. 9'
Angle of the Subſtylar with the Meridian 36. 58'
Difference of the Meridians 69. 27'

Angles of the Hour-lines, *with the* Meridian.

I.	II.	XI.	X.	IX.	VIII.	*Subſt.*	VII.	VI.	V.	IV.
D.M.	D.M.	D.M.	D.M	D.M.	D.M.	D.M	D.M.	D.M.	D.M	D.M.
28.25	78. 3	14.30	23.31	29.37	34.50	36.58	40. 8	45. 6	51.37	60.55

Parts of the Horizontal-line, *comprehended between the* Meridian *and the* Hour-lines.

I.	II.	XI.	X.	IX.	VIII	*Subſt.*	VII.	VI.	V.	IV.	
1245	7113	599	993	1309	1602	1732	1916	2309	2905	4136	

Parts of the Hour-lines, *comprehended between the Center of the* Dial, *and the* Equinoctial.

I.	XII.	XI.	X.	IX.	VIII	*Subſt.*	VII.	VI.	V.	IV	
7746	4039	3491	3321	3255	3230	3228	3232	3260	3336	3532	

Parts of the Equinoctial, *between the* Meridian *and the* Hour-lines.

I	XI	X	IX	VIII	*Subſt.*	VII	VI	V	IV		
4613	1100	1646	2012	2301	2428	2587	2882	3272	3861		

Angles of the Hour-lines, *with the* Equator *of the* Triangle *of* Signs.

I	XII	XI	X	IX	VIII	*Subſt.*	VII	VI	V	IV
D.M.	D.M.	D.M.	D.M.	D.M.	D M.	D.M.	D.M.	D.M.	D.M.	D.M.
23.10	49. 0	60.50	66.37	69.30	70.41	70.48	70.36	69.15	66. 2	59.40

Parts of the Hour-lines, *between the Center of the* Dial, *and the* Tropick *of* ♑.

I	XII	XI	X	IX	VIII	*Subſt.*	VII	VI	V	IV
3843	2931	2809	2796	2800	2803	2803	2803	2866	2796	2816

Parts of the Hour-lines, *between the Center of the* Dial, *and the* Tropick *of* ♋.

XII	XI	X	IX	VIII	*Subſt.*	VII	VI	V	IV
6494	4609	4090	3886	3811	3804	3816	3903	4135	4737

PROBLEM VIII.

How to deſcribe the Circles of Altitude, and the Verticals upon an Inclin'd Plain.

THE Circles of Altitude are repreſented upon an Inclin'd Plain, as the Parallels of the Sun upon an Horizontal Plain of the Oblique Sphere; and the Vertical Circles after the ſame manner as the Horary Circles upon the ſaid Horizontal Plain, taking the Zenith from the Center of the Horizontal Dial, and the Horizontal for the Equinoctial of the ſame Dial.

Wherefore if you divide into Degrees the Horizontal line, as we have divided the Equinoctial into Hours, and that thro' the Points of Diviſion you draw Right-lines to the Zenith, you will have the Repreſentation of the Vertical Circles: And if thoſe Right-lines be divided into Degrees, (as we have done the Hour-lines to mark upon them the Points of the Arches of the Signs) and all the Points of the ſame Degree be joyn'd by Curves, thoſe Curves will be the Circles of Altitude.

PROBLEM IX.

How to deſcribe the Circles of the Celeſtial Houſes in a Dial.

BEcauſe the Circles of the Celeſtial Houſes are great Circles, which interſecting in the Common Section of the Meridian and Horizon, divide the Equator, (according to ſome, and according to others the Firſt Vertical) into 12 equal Parts; it follows that if the Equinoctial or the Vertical be divided into Parts of 30 Degrees beginning from the Meridian, and thro' the Points of Diviſion Right-lines be drawn to the Section of the Horizontal and the Meridian, you will have the Repreſentation of the Circles of the Celeſtial Houſes.

This ſuppoſes the Dial to have an Horizontal Line; but if it has not one, as it happens in the Horizontal Dial, the Circles of the Celeſtial Houſes muſt be drawn by Right-lines parallel to one another and to the Meridian: And if the Dial has no Meridian Line as it happens in a Meridian Dial, thoſe Circles muſt be repreſented by Right-lines parallel to one another and to the Horizontal. There may happen other Caſes, which we ſhall not mention here, becauſe it is eaſy to reſolve them by our Principles, and thoſe Circles are of too little Conſequence to have any thing more ſaid concerning them.

FINIS.

To

To the READER

THO' *the Two Tables of* Page 69 *and* Page 70, *are only Calculated for the Latitude of* Paris; *yet they may be made to ſerve for any Latitude, by the following Method.*

For the Table of the Diſtance, *&c.* Page 69.

HAving, by Problem 6. Chap. 3 of the Second Part of the Geography, *found upon what Days the Sun is in thoſe Points of the Zodiack mark'd in the Firſt Column on the Left-Hand; you muſt*, by Probl. 16. of the ſaid Chap. *find out the Sun's Height for the Latitude of the Place where you are, upon the Hour requir'd of that Day, which anſwers to the Point of the Zodiack which you ſuppoſe the Sun to be in; then taking the Degrees of that Height from* 90 *Degrees, the Remainder will be the Sun's Diſtance from the Zenith inſtead of that in the Table.*

For the Table of the Verticals of the Sun, *&c.* Page 70.

HAving, by Probl. 6. Chap. 3. Geogr. Part 2, *found upon what Days the Sun enters into thoſe Signs of the Zodiack mark'd in the Firſt Column on the Left-Hand; you muſt*, by the ſaid Probl. 16. *find the Sun's Vertical for the Hour requir'd upon any of the ſaid Days, according to the Latitude of the Place where you are; and take the Diſtance of that Vertical from the Meridian inſtead of the Diſtance mark'd in the Table.*

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GNOMONICKS.

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CHAPTER I.

Of the Lemma's. 4

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the Angle CDF will be equal to the Angle CDG, or the Angle ADG to the Angle BDF. 8

LEM. V. Theor. *If the Semi-circles ABC and ADE be deſcrib'd about the ſide AC, and the Hypotenuſe AE of the Triangle ACE, Rectangular at C; and from the point F taken at pleaſure upon AC to the Two lines AC and AE, be drawn the Perpendiculars FB, FD; the Chords AB, AD, will be equal to one another.* 9

LEM. VI. Theor. *If the Two lines AB, CE, be perpendicular to the ſame line DH, and CD is equal to BC, and AF to AB; and at the point A the Angle HAG be made equal to the Angle ACB, by the line AG terminated at G by the line FG, perpendicular to DH; and that at the ſaid point A, the Angle HAI be made equal to the Angle CAE, and the line AH be made equal to the line AG; and laſt of all, if thro' the point I, IK be be drawn parallel to DH, and AK be joyn'd; the Angle HAK, will be equal to the Angle CDE.* 9

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FINIS.

Advertiſement.

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By *Robert Arnold.*